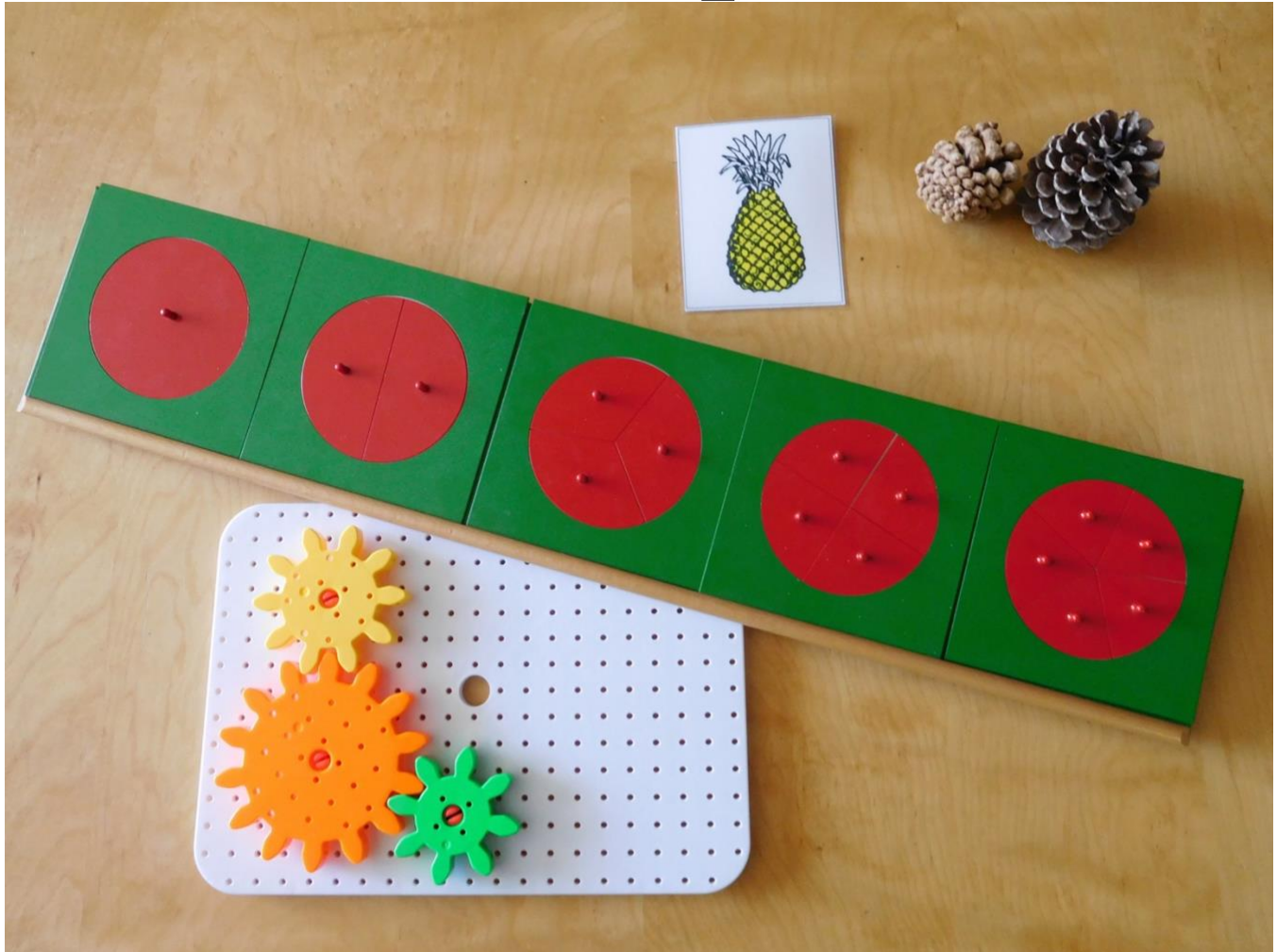


# Concrete Foundations for Ratio/Proportion



Third Edition

by Betsy A Lockhart

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## **Thoughts Before Beginning Ratio/Proportion**

Why ratios?

One obvious reason for studying ratios and proportions is that they are part of the common core standards beginning in sixth grade. We want our children to be successful with meeting or exceeding what is expected. But ratios (and rates) are a somewhat covert part of our everyday life: speed limits, hourly wages, price per pound, and recipe adjustments, to name a few. They are also an integral part of science, particularly physics. As such, ratio and proportion concepts are both an academic and a practical life application of fractions.

These lessons follow the type of sequence that Montessori philosophically advocated for all geometry and mathematics lessons:

- Introduce with a concrete/sensorial experience
- Integrate opportunities to apply using measurement and other real-life experiences
- Reason out the general rule / procedure / algorithm

This differs from traditional educational methods that often begin with the algorithm and then give applications in the form of story problems after the algorithm has been memorized.

Teachers should be aware that the lessons that follow are provided to guide the scope and sequence of instruction. Please do not assume that every child will need every lesson. Also, please also do not assume that by completing the follow-up activities that follow each lesson, every child will completely internalize all of the concepts and nuances of ratios and proportions. The degree to which children internalize concepts, processes, and algorithms will vary greatly depending upon their prior experience and their comfort with fractions, particularly with finding equivalent fractions and with fraction multiplication.

- Some children will intuit far more rapidly than these lessons progress. If children leap ahead, as long as they can explain their reasoning, they should be allowed to do so.
- If children show lack of confidence or confusion, they should have more opportunities to practice a given process than are provided in the included follow-ups. Traditional textbooks, workbooks, or worksheets can be a fine source for this supplemental practice as long as the teacher ensures that the resource is isolating the same difficulty as the lesson; remember that traditional sequences are often quite different from Montessori sequences – we don't want to give children worksheets that reveal an algorithm that has not yet been experienced or discovered!

Teachers should also be aware of precisely how children's knowledge is assessed on standardized tests. There is a plethora of ways to phrase and format questions involving ratios, rates, and proportions. It is our responsibility as educators to ensure that children do not encounter language or formats for the first time in an assessment situation. Something as simple as changing the way something is shown from a horizontal alignment to a vertical alignment can make what is well known feel quite foreign. If that happens, the test will not accurately reflect the child's understanding of the content. It is wise, therefore, to review any available test-preparation materials to evaluate what will be familiar and what suggests some targeted instruction or practice to help children accurately show what they know on assessments.

Please note that graphing is integral to these lessons. It is assumed that children have had prior experience with pictograms, bar graphs, and simple 2-coordinate graphing prior to these lessons. If this is not the case, please provide those experiences before the relevant lessons in this sequence.

Also note that there are graphs and other graphics provided throughout. These are provided to help teachers envision what the graph or graphic that they are creating in the lesson will look like when complete. Due to limitations of space / software, these graphs often lack elements that make a complete graph, for example:

- Title
- Key (if appropriate)
- Labels for the axes

For this reason, and because children benefit greatly from seeing graphs being created real-time, these graphs are not intended to be photocopied and shown to children unless the teacher adds the missing elements to make them complete. We do not want to model incomplete graphs for the children!

A natural cross-curricular extension to this topic is scale. Speaking to your art specialist about applications of ratio/proportions to scale drawings or inviting children to make scale replicas of something relevant to cultural topics might bear great fruit. (Think along the lines of recreating ancient architecture with sugar cubes, where each cube represents a certain number of cubits or feet...)

One final reason for incorporating ratios and proportions into the curriculum is that they keep fraction concepts and operations fresh! For whatever reason, it seems that fractions are tougher to internalize for most of us. More so than for other mathematical concepts, it seems that fractions cannot go for long periods of time without being used without significant atrophy. Bringing ratios and proportions into the sequence is a beautiful example of Montessori's spiral staircase in action. They refresh fraction concepts and operations and take them to a new level.

Convinced? Then let's get started!



# Ratio/Proportion

## Ratios: Concept, Simplification, and Equivalency

### Materials:

Objects to be used in first demonstration of ratio

*For the purpose of this discussion, we will use red and black checkers; however, if the items can be edible, like 2 varieties of wrapped candies or red and green grapes, it produces a more lasting memory for the children. The number of each item should be sufficient that each child in the lesson receives 3 of one and 4 of the other.*

A quantity of colored rectangles (“tape”) in at least 2 colors

Optional: copy of the recipe for Corn Dip (follows) or another taste treat

**Direct Aim:** to introduce the concept of ratios and relate them to fractions

**Indirect Aim:** further practice with fraction equivalencies

### Applicable Standards:

CCSS.MATH.CONTENT.6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

CCSS.MATH.CONTENT.6.RP.A.3 (partial – italicized standard not addressed here) Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, *double number line diagrams*, or equations.

CCSS.MATH.CONTENT.6.RP.A.3.A (partial–italicized standard not addressed here) Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, *and plot the pairs of values on the coordinate plane. Use tables to compare ratios.*

**Prerequisites:** significant success with fraction concepts, equivalencies, and simple operations

### Presentation:

#### Fraction Review

1. Review what children know about equivalent fractions and reducing fractions. Specifics will vary depending upon children’s previous experiences. The goal is to use “effortful retrieval” to engage the parts of the brain where the new knowledge will be stored.
2. Take them back even further – to first concepts. Ask them to describe what a fraction is as well as the significance of the numerator and denominator. Key points to draw from the children in their language:
  - A fraction is part of a whole (if children bring up improper fractions or mixed numbers, set them aside – we are discussing proper fractions only at this time)
  - The numerator tells the number of equal or equal-sized pieces
  - The denominator tells the number of those pieces that it takes to make a whole
  - The whole can be represented in many different ways: a red circle, a red square, an inch, a pie.... *Note: As is always the case in math, we define the unit to suit our needs. You may have used the phrase “One is what I say it is...” when introducing decimals, defining the million-cube from the wooden geometric hierarchy of materials to be 1. If so, that phrase will be useful here as well.*
  - A fraction is a number like 7 in that it doesn’t need units to have meaning. It can stand on its own (foreshadowing...)

### Fractions and Ratios

3. Using the fourths inset, lay out 3 of the 4 pieces and write the fraction for the children to see. Ask a volunteer to read the fraction. <“Three fourths.”>
4. The manipulative represents  $\frac{3}{4}$  of a circle. The written quantity could be  $\frac{3}{4}$  of a circle or  $\frac{3}{4}$  of the people in the class or just the pure number,  $\frac{3}{4}$ .
5. Explain that another way to read that expression would be, “Three out of four.”  
Draw 3 squares and a circle.  
“Three out of four shapes are squares.” “Three fourths of the shapes are squares.”  
They both say the same thing in slightly different ways.  
**They both tell the number of square shapes compared to the total number of shapes.**
6. We could also say that for every 3 squares there is 1 circle. Confirm that children see that relationship.  
**This tells the number of square shapes compared to the number of circular shapes.**
7. Add 3 triangles to the picture.  
Ask if it is still true that  $\frac{3}{4}$  of the shapes are squares? <It is not true. Now  $\frac{3}{7}$  of the shapes are squares.>  
Ask if it is still true that for every 3 square shapes there is 1 circular shape. <It is true.>
8. With the fraction  $\frac{3}{7}$ , we are expressing what part of all shapes are square: part to whole. With the second expression, “for every 3 squares there is 1 circle”, we are describing a relationship between 2 quantities. We don’t know nor do we need to know how many shapes there are in total. This expression is called a **ratio**.  
***A ratio compares two quantities to each other.***  
***A fraction compares a quantity to the whole.***
9. Place a specific number of black and red checkers (or other objects) into two piles on the table or rug. (*Per instructions for **materials** for this lesson: we will presume that there are 5 children in the lesson, so we have 15 red checkers in one pile and 20 black checkers in another pile.*) Indicating the piles of checkers, ask “How many?”  
< The children may reply that there are 2 piles OR that there are 35 checkers OR that there are 15 red checkers and 20 black checkers.> Accept all answers. These answers were absolute quantities. The children counted and reported how many there were.
10. How would we express the fraction of the checkers that are red? <15/35 or 3/7>  
How would we express the fraction of the checkers that are black? <20/35 or 4/7>  
These answers told us how many of one type there are compared to the total of all checkers. That’s what fractions do – they say how much relative to the whole.
11. NOW... what if we wanted to express the relationship between the red checkers to black checkers without mentioning how many there are in total? We would say that there are **15 red checkers TO 20 black checkers**. This expression is a ratio. Remember, **a ratio expresses how many there are of one thing relative to another thing**. Ratios are all about relationships.

12. There are 3 ways to write this ratio:

looking like a fraction:  $\frac{15}{20}$  with a colon: **15:20** with words: **15 to 20**

Each of these ratios would be read. “15 to 20” or “15 out of 20”.

In context, no matter how we wrote the ratio, we would say, “The ratio of red checkers to black checkers is 15 to 20.”

*OPTIONAL – discuss why, when writing a ratio written as a fraction, it is important to also provide enough language that the reader knows that it is not a fraction. Here, if we say that fifteen thirty-fifths of the checkers are red, we know that the total number of checkers is 35 or a perfect multiple of 35. We would express the ratio of red checkers to black checkers by saying that for every 15 red checkers there are 20 black checkers.*

13. Next, ask the children how they would write the ratio of black checkers to red checkers. < 20/15 OR 20:15 OR 20 to 15 > Order matters! The order of the numbers must match the order of the words.

14. Challenge the children to complete the following:

The ratio of red checkers to the total: < 15:35 **or** 15/35 **or** 15 to 35 >

The ratio of black checkers to the total: < 20:35 **or** 20/35 **or** 20 to 35 >

The ratio of red checkers to black checkers: < 15:20 **or** 15/20 **or** 15 to 20 >

What fraction of the checkers are black? < Four sevenths or four out of seven >

*Note: all fractions are ratios but not all ratios are fractions.*

*Whenever we ask for the ratio of part to whole, like we did when we asked for the ratio of red checkers to the total, the resulting ratio is also a fraction. In this case, the ratio of red checkers to the total is fifteen to thirty-five AND 15/35 of the checkers are red.*

*In some circles, they refer to there being 2 kinds of ratios: part-to-part ratios and part-to-whole ratios. Part-to-whole ratios are fractions.*

15. Write the ratio  $\frac{15}{35}$ . *In our current example, it is both a fraction and a ratio.*

35 “Fifteen thirty-fifths of the checkers are red. Fifteen out of thirty-five checkers are red.” Both express the number of red checkers compared to the number of all checkers – the whole.

16. Ask the children if they notice anything about the fraction 15/35. < It can be reduced to 3/7.> Ask what the children did mathematically to reduce 15/35 to 3/7. <They divided numerator and denominator by 5.>

Ask what could be done to the checkers to represent that action. <Make 5 identical sets of checkers.> *Each of the 5 identical sets would be comprised of a group of 3 black checkers and a group of 4 red checkers.*

Since there are 5 children in the lesson (what a coincidence!), each child can receive a set.

17. Ask these questions:

In your set, what is the ratio of red checkers to black checkers? < 3 to 4 >

Is that the same relationship as in every other set? <Yes>

In each set, what is the ratio red checkers to the total: < 3:7 >

Is that equivalent to the ratio of total red checkers to total, 15:35? <Yes!>

In each set, what is the ratio black checkers to the total: < 4:7 >

Is that equivalent to the ratio of total red checkers to total, 20:35? <Yes!>



Sample Lesson from Ratio/Proportion (Lockhart 2020)

In each set, what fraction of the checkers are red? < Three sevenths or three out of seven >  
In each set, what fraction of the checkers are black? < Four sevenths or four out of seven >  
When we had all of the checkers together, what fraction of the checkers were black?  
< 20/35 or 4/7: Four sevenths or four out of seven >

*Note: be sure that children see that it is equally true that 20 of the 35 are black AND that 3 out of every 4 are black. The latter is just a simplified (reduced) form of the ratio. The ratios are same for the 5 identical small sets as for the whole group of checkers.*

Expressing Ratios

18. When we express ratios, it is ok to report the comparison absolutely – writing the exact numbers in each set. But it is often easier to understand if we reduce the ratio to simplest terms just like we do for fractions.

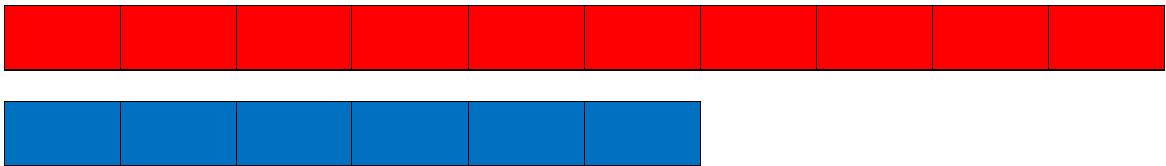
*If the objects used were edibles rather than checkers, give the children something to put the edibles in until it is time to eat and have them set the edibles aside.*

**→ For the rest of this lesson, we will annotate ratios using a colon exclusively. ←**

19. Another way to represent ratios is with a **tape diagram**. Here, colored rectangles are laid out like units on a tape measure. They are arranged to represent a ratio.

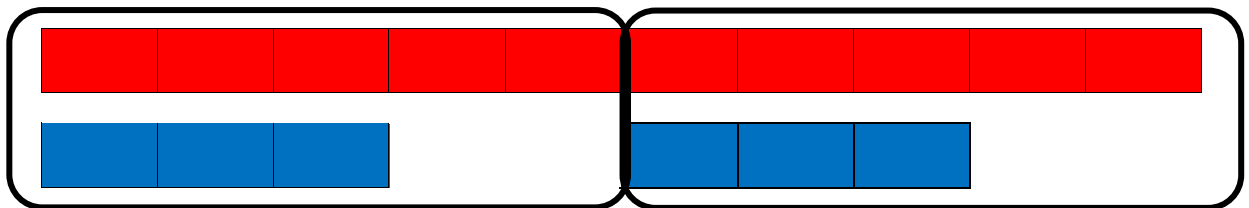
Lay out 10 red rectangles in a row and 6 blue rectangles in a separate row, as shown, to illustrate the results of an imaginary vote:

*Sixteen children were planning an overnight camping trip; 10 wanted to camp in the mountains and 6 preferred to camp by the river.*



We can look at this tape diagram and easily see the ratio of the results. Ask the children for the ratio of those favoring the mountains to those who prefer the river. <10:6>.

We can re-arrange the tape to show the ratio in its reduced form, 5:3. We split the tape into two identical sets, which means each set will have half as many members as the original ratio.



Discuss the similarities between reducing ratios and reducing fractions.

20. Ask the children to take turns coming up with other ratios that are equivalent to 5:3.  
<Answers might include 10:6, 15:9, 20:12, 25:15... 50:30 ... 100:60... etc.>

Sample Lesson from Ratio/Proportion (Lockhart 2020)

21. Ask the children to use their knowledge of equivalent fractions to complete the list of equivalent ratios:

$$\begin{array}{l} 16:12 \\ \underline{\quad} : 3 \quad < 4 > \\ 8 : \underline{\quad} \quad < 6 > \\ 32 : \underline{\quad} \quad < 24 > \text{ etc.} \end{array}$$

Continue until children show confidence. Ask the children to draw a tape diagram to illustrate one of the above ratios.

22. Pose a situation, drawing a sketch to support the story. *Eli was working as a parking lot attendant at a lot near the airport. He started to think about how often the lot was full – they had to turn customers away. But the lot wasn't really as full as it seemed: some of the spaces were occupied by motorcycles that need very little space – a lot of space was wasted. To study this more, he decided he needed to know if there were enough motorcycles to make a difference; he needed to know the ratio of motorcycles to larger vehicles. He decided to count the number of cars, trucks, and motorcycles that came into the lot on his shift.*

*During his first shift, he counted 240 cars, 180 trucks, and 60 motorcycles.*

What is the ratio of motorcycles to cars?  $< 60:240 \text{ or } 1:4 >$

What is the ratio of motorcycles to trucks?  $< 60:180 \text{ or } 1:3 >$

What fraction of all vehicles were motorcycles, cars, or trucks?

Motorcycles:  $< 60/480 \text{ or } 1/8 >$

Cars:  $< 240/480 \text{ or } 1/2 >$

Trucks:  $< 180/480 \text{ or } 3/8 >$

23. Another way that ratios are used is in sizing up or sizing down a recipe. Here is a recipe for Corn Dip (add chips or raw veggies for dipping and eat!).

3	cans Mexican-style corn	1	jalapeno pepper, chopped
1	can diced green chilis, drained	3/4	cup mayonnaise
5	green onions, chopped	10	ounces shredded Cheddar cheese
8	ounces sour cream		

If you are having a big party, you may want more corn dip than this recipe makes. You can do that and have it taste the same as long as you keep the ratios of ingredients the same.

Let's say that you decide that you need 3 bowls of dip. The ratio of ingredients that you will need to ingredients needed to those called for in this recipe will be 3:1.

Ask the children how much they will need of each ingredient.

9	cans Mexican-style corn	3	jalapeno pepper, chopped
3	cans diced green chilis, drained	2¼	cup mayonnaise
15	green onions, chopped	30	ounces shredded cheddar cheese
24	ounces sour cream		

We can check our math by seeing if the ratio of ingredients are the same in each of the two versions of the recipe.

Verify:

the ratio of cans of corn to cans of chilis is the same.  $\langle 3:1 = 9:3 \rangle$

the ratio of ounces of cheddar cheese to ounces of sour cream is the same.  $\langle 30:24 = 10:8 \rangle$

The ratio of the number of onions to the number of jalapenos is the same  $\langle 5:1 = 15:3 \rangle$

For all of the above ratios, the units were the same on both sides of the ratio. We were comparing **cans** of corn to **cans** of chilis, **ounces** of cheese to **ounces** of sour cream, etc.

For these, we can shorten our notation:

The ratio of corn to chilis is 3:1

The ratio of cheddar cheese to sour cream is 5:4

The ratio of onions to jalapenos is 5:1

***A ratio compares two quantities of the same type of object or measure.***  
(*cups:cups, checkers:checkers, etc.*)

OPTIONAL – Change the story: halve the recipe using a ratio of  $\frac{1}{2}:1$ . Create a new list of ingredients and spot check a few ratios.

**Point of interest:**

1. Similarities to fractions – the children already know lots about ratios!

2. There are three ways to write ratios:

looking like a fraction:  $\frac{15}{20}$       with a colon: **15:20**      with words: **15 to 20**

3. There are three ways to express ratios verbally:

15 to 20      15 for each/every 20      15 out of every 20

**Follow-up:** Children should gain familiarity with ratios and equivalent ratios.

**Control of Error:** The checker (follows)

## **World Famous Corn Dip**

### **Ingredients**

3 cans Mexican-style corn  
1 can diced green chilis, drained  
5 green onions, chopped  
8 ounces sour cream  
1 jalapeno pepper, chopped  
3/4 cup mayonnaise  
10 ounces shredded cheddar cheese

Chop the green onions and jalapeno.  
To the onions and jalapeno, add the corn and chilis

In a separate bowl, combine the sour cream and mayonnaise. Mix well.

Add the cheddar cheese. Stir until well blended.  
Add the corn mixture and stir thoroughly.

Serve with corn chips or raw vegetables that are suitable for dipping (zucchini, jicama, red/green/yellow peppers, etc.)

Enjoy!

## Ratios: Concept, Simplification, and Equivalency

In your own words, please define or define the following:

ratio: \_\_\_\_\_

fraction: \_\_\_\_\_

\*\*\*\*\*

**Ratios** At the dog park one day, there were 10 black dogs, 5 brown dogs, 3 white dogs, 12 multicolor dogs, and 2 white llamas that thought they were dogs. (Total 30 dogs, 32 animals) Please write the ratio using a colon (6:7) for each of the following. Reduce if possible:

Ratio: black dogs to brown dogs \_\_\_\_\_ Ratio: white dogs to all dogs \_\_\_\_\_

Ratio: dogs to llamas \_\_\_\_\_ Fraction of dogs that are black \_\_\_\_\_

Ratio: legs to ears \_\_\_\_\_ Fraction of animals that are white \_\_\_\_\_

For every brown dog, there 2 \_\_\_\_\_ dogs. There are \_\_\_\_\_ dogs for each llama.

\*\*\*\*\*

Shade in all of the squares that have a ratio equivalent to 3:5. The bottom row is BONUS!  
HINT: Work smart! Some of these are OBVIOUSLY not equivalent to 3:5. Remember that order matters!

15:6	4:10	51:85	33:55	1:5	30:50	90:150	10:6	45:20
2:10	27:45	42:70	31:15	3:5	25:1	15:25	36:60	15:12
30:55	18:30	12:15	8:21	6:33	17:23	13:21	21:35	75:100
13:18	24:40	22:26	8:12	24:40	5:10	10:5	39:65	5:10
21:33	27:40	45:75	2:10	15:3	34:17	6:10	4:6	33:11
5:3	17:21	7:8	51:85	13:15	15:25	55:33	25:13	19:42
17:35	3:15	13:43	12:20	48:75	9:15	53:55	9:20	10:6
10:6	6:4	7:31	23:23	48:80	7:13	4:2	17:19	20:50
$\frac{1:1}{2}$	$\frac{3:5}{8 \ 8}$	$\frac{3:5}{15 \ 25}$	$\frac{1:1}{15 \ 25}$	$\frac{3:1}{5}$	$\frac{1:1}{3 \ 5}$	$.75 : \frac{3}{4}$	$\frac{15:15}{5 \ 3}$	$\frac{3:5}{3 \ 5}$

Draw a picture showing a ratio of 4:6 in a box on the left. Draw a picture showing a ratio of 6:4 in a box on the left. (Example: 4 oranges to 6 apples and 6 stars to 4 moons). Circle or group the objects on each side to show that 4:6 is equivalent to 2:3 and 6:4 is equivalent to 3:2.

\*\*\*\*\*

Use a tape diagram to show that 15:3 is the same as 5:1

\*\*\*\*\*

Below is the recipe for Toll House Chocolate Chip Cookies. It makes 10 dozen cookies – that’s a LOT of cookies! You have only one egg and can’t get to the store. You have a friend coming over in an hour. Luckily, you know that as long as you keep the ratios the same, the cookies will taste the same. Using ratios, convert this recipe.

What is the ratio of eggs required for a full recipe to eggs that you have? \_\_\_\_\_

Full Recipe

- 1 ½ (or 3/2) cup sugar
- 1 ½ (or 3/2) cup brown sugar
- 4 ½ (or 9/2) cup flour
- 2 teaspoons baking soda
- 2 teaspoons salt
- 4 sticks butter
- 2 teaspoons vanilla extract
- 4 large eggs
- 4 cups chocolate chips
- 2 cups chopped nuts

Your Recipe

- \_\_\_\_\_ cup sugar
- \_\_\_\_\_ cup brown sugar
- \_\_\_\_\_ cup flour
- \_\_\_\_\_ teaspoon baking soda
- \_\_\_\_\_ teaspoon salt
- \_\_\_\_\_ sticks butter
- \_\_\_\_\_ teaspoon vanilla extract
- \_\_\_\_\_ large eggs
- \_\_\_\_\_ cups chocolate chips
- \_\_\_\_\_ cup chopped nut

**Double check your numbers!**

What is the ratio (improper fraction) of flour to all sugar in the original recipe? \_\_\_\_\_

What is the ratio of flour (improper fraction) to all sugar in your recipe? \_\_\_\_\_

## Control of Error for Ratios: Concept, Simplification, and Equivalency

In your own words, please define or define the following:

ratio: Answers will vary but should say something about showing a relationship between 2 quantities

fraction: Answers will vary but should say something about showing parts of a whole or part to whole

\*\*\*\*\*

**Ratios** At the dog park one day, there were 10 black dogs, 5 brown dogs, 3 white dogs, 12 multicolor dogs, and 2 white llamas that thought they were dogs. (Total 30 dogs, 32 animals) Please write the ratio using a colon (6:7) for each of the following. Reduce if possible:

Ratio: black dogs to brown dogs 10:5 or 2:1

Ratio: white dogs to all dogs 3:30 or 1:10

Ratio: dogs to llamas 30:2 or 15:1

Fraction of dogs that are black 10/30 or 1/3

Ratio: legs to ears 128:64 or 4:2 or 2:1

Fraction of animals that are white 5/32

For every brown dog, there are 2 black dogs.

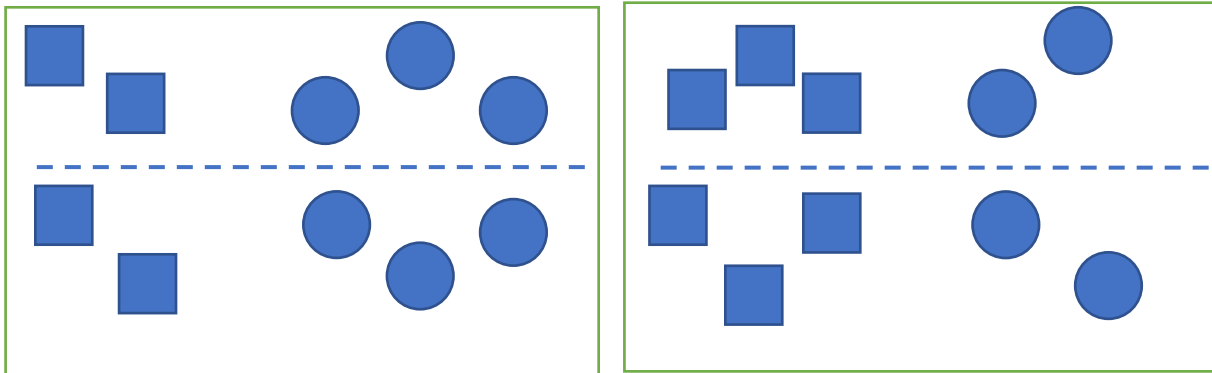
There are 15 dogs for each llama.

\*\*\*\*\*

Shade in all of the squares that have a ratio equivalent to 3:5.

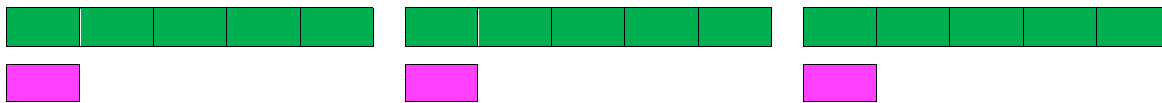
6:15	4:10	51:85	33:55	1:5	30:50	90:150	10:6	45:20
2:10	27:45	42:70	31:15	3:5	25:1	15:25	36:60	15:12
30:55	18:30	12:15	8:21	6:33	17:23	13:21	21:35	75:100
13:18	24:40	22:26	8:12	24:40	5:10	10:5	39:65	5:10
21:33	27:40	45:75	2:10	15:3	34:17	6:10	4:6	33:11
5:3	17:21	7:8	51:85	13:15	15:25	55:33	25:13	19:42
17:35	3:15	13:43	12:20	48:75	9:15	53:55	9:20	10:6
10:6	6:4	7:31	23:23	48:80	7:13	4:2	17:19	20:50
$\frac{1:1}{2}$	$\frac{3:5}{8:8}$	$\frac{3:5}{15:25}$	$\frac{1:1}{15:25}$	$\frac{3:1}{5}$	$\frac{1:1}{3:5}$	$.75 : \frac{3}{4}$	$\frac{15:15}{5:3}$	$\frac{3:5}{3:5}$

Draw a picture showing a ratio of 4:6 in the box on the left. Draw a picture in the rightmost box that shows a ratio of 6:4. (Example: 4 oranges to 6 apples and 6 stars to 4 moons). Circle or group the objects on each side to show that 4:6 is equivalent to 2:3 and 6:4 is equivalent to 3:2. *Sample answer:*



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Draw a tape diagram to show that 15:3 is the same as 5:1



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Below is the recipe for Toll House Chocolate Chip Cookies. It makes 10 dozen cookies – that’s a LOT of cookies! You have only one egg and can’t get to the store. You have a friend coming over in an hour. Luckily, you know that as long as you keep the ratios the same, the cookies will taste the same. Using ratios, convert this recipe.

What is the ratio of eggs required for a full recipe to eggs that you have? 4:1

Full Recipe

- 1 ½ (or 3/2) cup sugar
- 1 ½ (or 3/2) cup brown sugar
- 4 ½ (or 9/2) cup flour
- 2 teaspoons baking soda
- 2 teaspoons salt
- 4 sticks butter
- 2 teaspoons vanilla extract
- 4 large eggs
- 4 cups chocolate chips
- 2 cups chopped nuts

Your Recipe

- 3/8 cup sugar
- 3/8 cup brown sugar
- 1 1/8 cup flour
- 1/2 teaspoon baking soda
- 1/2 teaspoon salt
- 1 stick butter
- 1/2 teaspoon vanilla extract
- 1 large egg
- 1 cup chocolate chips
- 1/2 cup chopped nut

**Double check your numbers!**

What is the ratio (improper fraction) of flour to all sugar in the original recipe? 9/2 : 3/2

What is the ratio of flour (improper fraction) to all sugar in your recipe? 9/8 : 3/8