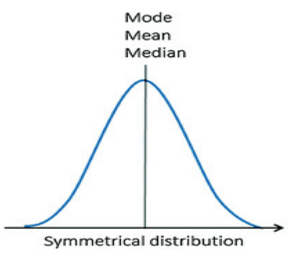
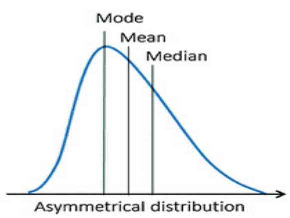
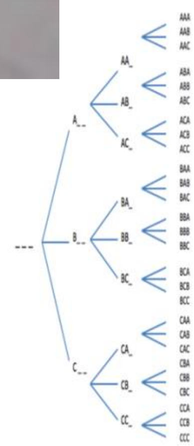
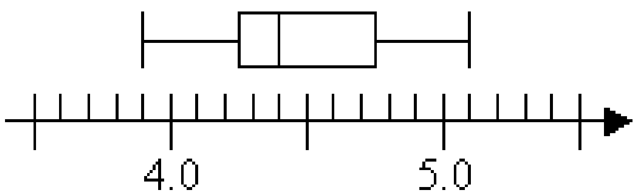
Concrete Foundations for
Probability/Statistics









Third Edition

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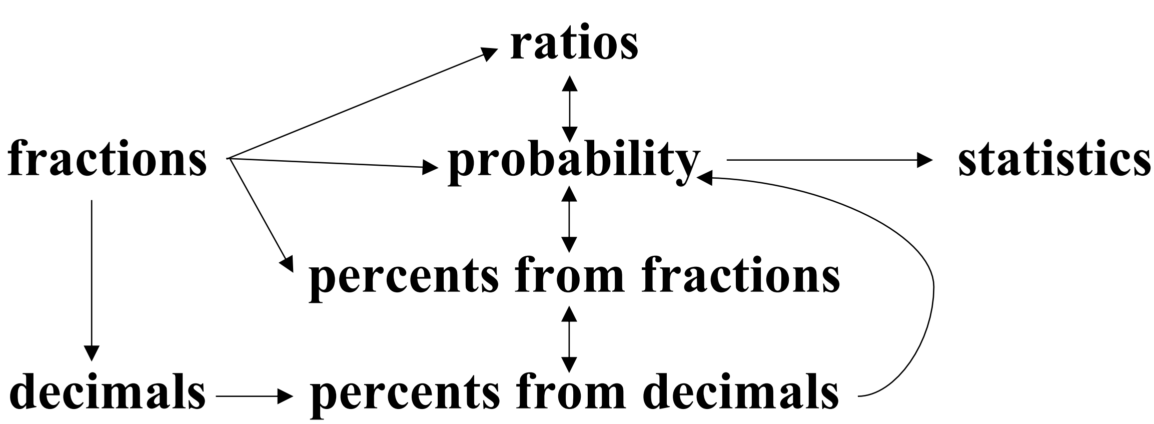
Thoughts Before Beginning Probability

Probability is nothing more than an extension of what children learned while studying ratios. We have used the term probability prior to this point in the lesson sequence, particularly around the probability of precipitation on a given day, in lessons on percents. It is a statement about how likely something is to happen. A probability analysis provides a number (expressed as a ratio or a percent) that helps us better understand the likelihood of a particular outcome of an event that is fundamentally random.

Generally speaking, probability appeals to children with mathematical acumen in a wide variety of areas. It requires little computationally, is highly logical, and is highly relatable. As such, it can be introduced in a variety of different places within the math sequence:

- using fraction notation anywhere in the sequence after children are comfortable with reducing common fractions, as an application of that skill
- using both the fractional and percent notation anywhere in the sequence after children are comfortable converting common fractions to decimal fractions, as an application of that skill
- as a precursor to the study of statistics, as is shown in this album. Probability and Statistics are two topics that are so often linked that they are sometimes voiced almost as a single thought: *Probabilityandstatistics.* In a sense, statistics (yet to come) is the analysis of events that are governed by probability, but the two topics can be studied in either sequence.

Common Core Standards expect little or nothing from children until Grade 7, although tree diagrams (see the second lesson in this series) can appear on standardized tests. (See standards 7.SP.C.7 AND 7.SP.C.8). Common Core actually holds children accountable for concepts related to statistics before those related to probability. Since a very basic understanding of probability enhances understanding of statistics, it is placed before statistics in this body of work. It is left to teachers' discretion, using understanding of students' skills and abilities relative to standards / accountability, and of course student interest, to determine the sequence to implement.



Probability
The Concept of Probability

Materials:

Large tickets with the words *probably, probable,* and *probability* and their definitions (copies follow the lesson)
Red and black checkers
Stamp Game
Math notebook or scrap paper and pencil for each child
A small supply of coins (for follow-up)
A deck of 52 playing cards (for follow-up)

Direct Aim:

Explore the concept of probability concretely

Indirect Aim:

Relate previously studied concepts to one another to produce a richer understanding of mathematics as a whole. Previously studied concepts might include:

- reducing fractions
- converting fractions to percents
- ratios

Note: modify presentations and provided follow-ups to match children's prior experiences

Prerequisites:

As noted in Thoughts Before Beginning Probability (previous page) the only true prerequisite is the ability to understand equivalent fractions and to reduce fractions to lowest terms. The lesson as written assumes that children have prior experience with converting fractions to percents and with ratios. If these lessons on Probability are presented earlier in the sequence than shown here, please adjust accordingly

Presentation:

1. Place the word ticket *probably* on the rug. Invite the children to use the word in a sentence. After a few examples, ask the children to define the word. When they have crafted a definition that most feel closely represents the meaning of the word, show the dictionary definition. Discuss any similarities or differences in the two definitions.

2. Repeat this process with the words *probable* and *probability.* (We began with *probably* because it is the children's vernacular language – the word *probable*, although the root, is far less familiar.)

3. Ask the children which of the three words is most likely to be used in mathematics – which is most likely to be represented by a number. <*Probability*> Probability reflects the idea that we can use numbers to tell us how likely something is to happen or to be true.

4. Illustrate the concept and the notation of probability as being a ratio of the number of possibilities that meet a given outcome or condition to the total number of possible outcomes: P(outcome) = $\frac{\text{\# of outcomes that match the criteria}}{\text{\# of possible outcomes}}$

Show	Desired outcome	Number of desired outcomes	Total number of outcomes	Probability (and recording)
1 red checker & 1 black checker	red checker	1	2	P(red) = $\frac{1}{2}$ "The probability is 1 in 2."
3 blue stamps & 5 red stamps	red stamp	5	8	P(red) = $\frac{5}{8}$ "The probability is 5 in 8."

5. Illustrate the convention that probabilities are represented in lowest terms, like fractions:

Show	Desired outcome	Number of desired outcomes	Total number of outcomes	Probability (and recording)
3 red checkers & 6 black checkers	red checker	3	6	P(red) = $\frac{3}{6} = \frac{1}{2}$ "The probability is 1 in 2."
9 blue stamps & 15 red stamps	red stamp	9	15	P(red) = $\frac{9}{15} = \frac{3}{5}$ "The probability is 3 in 5."

6. Now comes the fun part! Place equal amounts of red and black checkers in a bag and shake well. (For the purpose of this write-up, we will assume that there are 8 of each color.) One by one, invite each child to come forward to draw a checker out of the bag. The goal each time is to draw out a black checker. Before each draw, ask the group to identify the probability of success– the probability that the child will draw a black checker. At the end of the draw, record the result and ask the children to determine the ratio of successful draws to total draws. **Each time, return the chosen checker to the bag so that each time a draw is made from a bag with equal numbers of red and black checkers.** At the conclusion of each draw, note whether the actual ratio is greater than, less than, or equal to the theoretical probability of success (which is always 1/2).

   Shown below is an imaginary result. The goal is to help the children discover that probability has 2 meanings: the chance of a favorable outcome on a single draw AND the projected ratio if the experiment were repeated an infinite number of times. The more times that the experiment is repeated, the closer the ratio will be to the probability of a single draw.

Note: if there are few children in the lesson (<8) give each child 2-3 turns, enough that there are at least a dozen draws.

Draw #	P(black)	Outcome	# of successful outcomes	Ratio successful draws:total draws
1	$\frac{1}{2}$	red	0	$\frac{0}{1}$
2	$\frac{1}{2}$	black	1	$\frac{1}{2}$
3	$\frac{1}{2}$	black	2	$\frac{2}{3}$
4	$\frac{1}{2}$	black	3	$\frac{3}{4}$
5	$\frac{1}{2}$	red	3	$\frac{3}{5}$
6	$\frac{1}{2}$	black	4	$\frac{4}{6} = \frac{2}{3}$
7	$\frac{1}{2}$	red	4	$\frac{4}{7}$
8	$\frac{1}{2}$	red	4	$\frac{4}{8} = \frac{1}{2}$
…	$\frac{1}{2}$	…	…	…

7. Reiterate the point that while the probability of drawing a black checker is 1/2 for each draw, the actual ratio of black checkers drawn to the total number of draws will not always be 1/2! It is not even possible if we have an odd number of draws! If the ratio of black checkers drawn to total draws were exactly 1/2 every time, if we had drawn 7 times, 3 ½ draws would be black. "What would that even mean?"

   With a probability of 1/2, if the draws are completely fair, if we repeat the experiment an infinite number of times, the ratio will also be 1/2. Lead the children to the conclusion that the more times the experiment is repeated, the more likely the ratio will be exactly or very close to 1/2.

8. Explore probability when there are more than 2 items in the full set. From the stamp game, select 3 green stamps, 4 blue stamps, and 5 red stamps and mix them on the rug. Ask each child to write the equation and solution for the probability of drawing a green stamp, as a means of assessing whether everyone is ready for the next concept. < P(green) = 3/12 or 1/4>
   Ask the students to write the equation and solution for the probability of drawing a blue stamp and then for the probability of drawing a red stamp. <P(blue) = 4/12 or 1/3 and P(red) = 5/12>
   If children are participating with confidence, continue onto the next step. If not, end the lesson and give the children an opportunity to practice what they have learned.

9. With the mix of stamps from step 8, ask the children to discuss the probability of drawing a stamp. "If I put these stamps into an otherwise empty bag, put my hand in the bag, and pulled something out, what would the probability be of my drawing out a stamp?" Allow the discussion to evolve until they conclude that probability is 12/12 or 1. We would call a probability that is equivalent to 1 *certainty.* Explain that if we only put stamps into the bag, we can only draw out a stamp. There is no other possibility. Place the appropriate word and definition tickets on the rug.
   We would write that probability P(green or blue or red). We can find the solution two ways:
   - The first way adds the 3 individual probabilities. Ask the children to add the three fractions representing each of the 3 probabilities.
     <P(green or blue or red) = 3/12 + 4/12 + 5/12 = 12/12 = 1>
   - The second way is to go back to our original definition of probability:
     $$P(\text{outcome}) = \frac{\text{\# of outcomes that match the criteria}}{\text{\# of possible outcomes}}$$
     How many stamps are either green or blue or red? <12>
     How many stamps are there in total? <12>
     Probability (green or blue or red) = 12/12 = 1. Certainty.

10. With the mix of stamps from step 8, ask the children to discuss the probability of drawing a checker. "If I put these stamps into an otherwise empty bag, put my hand in the bag, and pulled something out, what would the probability be of my drawing out a checker?" Allow the discussion to evolve until they conclude that probability is 0/12 or 0. We would call a probability that is equivalent to 0 *impossibility.* Reiterate that if we only put stamps into the bag, we can only draw out a stamp. There is no other possibility. Place the appropriate word and definition tickets on the rug.

11. A probability analysis provides a number (expressed as a ratio or a percent) that helps us better understand the likelihood of a particular outcome of an event that is fundamentally random. Draw the children's attention to the range of probabilities:

impossible	possible	certain
P(success) = 0	0<P(success)<1	P(success) = 1

12. What do we know if the probability of a particular event happening is 1/2? Let's say that we were studying on-time departures from a particular airport. P(on time) = 1/2
    - What fraction of flights leave on time? <1/2>
    - What fraction of flights leave late? <1/2>
    - Is that a good record? <NO!>

    Here are some ways that we can express our findings: (Lay out corresponding tickets.)
    - 1 out of every 2 flights leaves on time
    - A flight is equally likely to be on-time as to be late.
    - The flight has even odds of being on time
    - A flight has a 1 in 2 chance of being on-time
    - Half of the flights depart on time

    Point out that some of these statements are talking about the probability of one flight leaving on time. Others are talking about flights in general – if an infinite number of flights were cataloged, half of them would be on time, reinforcing the dual meaning of probability.

13. What could we say if the study were repeated a year later and found that P(on time) was 3/4 rather than 1/2? (Lay out the ticket reading P(success) = 3/4). Ask the following:
    - "Has the on-time fate improved, gotten worse, or stayed the same?" <It is better!>
    - "What are some ways that we can express our findings?" (Lay out corresponding tickets as children offer expressions. Be prepared with blank tickets in case they offer unanticipated correct expressions!)
        - 3/4 of all flights leave on time; flights are on time 3/4 of the time.
        - 1/4 of all flights leave late; flights are late 1/4 of the time.
        - 3 out of 4 flights leave on time
        - A flight is more likely to be on time than to be late (children might say that the odds are uneven, patterning their response on those of the prior scenario)
        - A flight has a 3 in 4 chance of being on time

NOTE: if children have had extensive experience with ratios, much of this lesson may have been self-evident. If children seem eager to get to newer concepts, <u>and if the teacher is **certain** that all have a firm grasp of what has been presented so far</u>, segueing directly into the next lesson is acceptable. However, please realize that to get full benefit from the following lesson, children need to have the concepts from this lesson *well* in hand – the next lesson is pretty heady by comparison. For this reason, if combining the two lessons, it is highly suggested that children do both the follow-up activity for this lesson and the next, to further ensure that the fundamentals are well in hand. This way, if a child is less than 100% successful with the follow-up, it will be easier to discover which concepts need further edification and practice.

Follow-up: Children should explore probability through concrete experiments and abstract recording (sample follows). Note that on this follow-up, there is an activity that invites children to work with a partner. A classroom leadership best-practice is to suggest that children wishing to partner determine a fair way to share the tosses before beginning (i.e. one tosses 5 times while the other records and then they change jobs). If you prefer to not offer collaboration as an option, white-out that line before copying for the children.

CLASSROOM LEADERSHIP TIP: Generally speaking, it is not advised that children do this activity with more than a single partner; groups of 3 or larger tend to involve a lot of waiting-time, which inevitably leads to distraction. Of course, special circumstances may dictate allowing a larger group, such as when there are only 3 children receiving the lesson.

Control of Error: Checker card (follows) or teacher.

probably	*adverb* almost certainty
probable	*adjective* likely to be the case or to happen
probability	*noun* the likelihood of something happening
	P(success) = $\frac{\text{\# of outcomes that match criteria}}{\text{\# of possible outcomes}}$
certainty	P(success) = 1
impossibility	P(success) = 0
P(success) = $\frac{1}{2}$	1 out of 2

equally likely	even odds
1 in 2 chance	half are successful
50/50	P(success) = $\frac{3}{4}$
more likely than not	3 in 4 chance
three fourths are successful	3 out of 4
0% probability	50% probability
75% probability	100% probability

The Concept of Probability

Please define *probability* in your own words: ____

____

If you read the expression P(success) = 4/5, what would you know? What are at least 3 ways that you could express this relationship? ____

____

Please get a coin. Answer the following questions and then conduct the experiment.
If this is a fair coin, and if a fair toss is one where the coin lands flat (not on an angle or on an edge), how many possible outcomes are there for a single toss? ________________________

For a single flip, what is:
P(heads)? _____ P(tails)? _____ P(heads or tails)? _____ P(heads and tails)? _____

Activity One:
Create this chart in your math book. Flip your coin as many times as you like in 5 minutes or less. Before each flip, record the P(tails) for each flip. After each flip, record the outcome, the number of successful outcomes in the experiment so far, and the ratio of successful draws to total draws so far
You may do this activity with a friend or on your own.

Flip #	P(tails)	Outcome	Total # of successful outcomes so far	Ratio successful flips:total flips so far
1				
2				
3				
…	…	…	…	…

Highlight any rows where the ratio of successful flips (tails) to the total number of flips matches the P(tails). THINKING QUESTION: How can you explain the rows that are not highlighted?

Why do they not match? ____

____

____

Activity Two: In a deck of 52 playing cards (no jokers), how many cards are:

red? ______ black? ______ clubs?_______ diamonds?_______ hearts? ______ spades______

If you drew a card at random, what is P(red)? _______ P(heart)? _______ P(9 of hearts)? _______

BONUS: Design an experiment to test P(red) or P(heart)!

Control of Error for The Concept of Probability

Please define *probability* in your own words: answers will vary, but should say something about defining the **likelihood** that something will happen or **likelihood** of a particular outcome. If your answer also said that probability is a **number**, that is extra-correct!

If you read the expression P(success) = 4/5, what would you know? What are at least 3 ways that you could express this relationship? Answers will vary, but might include the following: **4 out of 5** trials will be successful, **4/5** are likely to be successful, more likely than not to be successful, **4 in 5** and potentially others. If in doubt about your answer, please check in!

Please get a coin. Answer the following questions and then conduct the experiment.
If this is a fair coin, and if a fair toss is one where the coin lands flat (not on an angle or on an edge), how many possible outcomes are there for a single toss? 2 outcomes: heads or tails.

For a single flip, what is:
P(heads)? 1/2 P(tails)? 1/2 P(heads or tails)? 2/2 or 1 P(heads and tails)? 0/2 or 0

Activity One:
Create this chart in your math book. Flip your coin as many times as you like in 5 minutes or less. Before each flip, record the P(tails) for each flip. After each flip, record the outcome, the number of successful outcomes in the experiment so far, and the ratio of successful draws to total draws so far
You may do this activity with a friend or on your own.

Flip #	P(tails)	Outcome	# of successful outcomes	Ratio successful flips:total flips
1	$\frac{1}{2}$	Answers vary	Answers vary	Answers vary
2	$\frac{1}{2}$	Answers vary	Answers vary	Answers vary
3	$\frac{1}{2}$	Answers vary	Answers vary	Answers vary
…	…	…	…	…

Highlight any rows where the ratio of successful flips (tails) to the total number of flips matches the P(tails). THINKING QUESTION: How can you explain the rows that are not highlighted? Why do they not match? Answers will vary. Probability is based on the idea that if we repeated the experiment an infinite number of times, it WOULD calculate out to exactly ½. With fewer repetitions of the experiment, we might get 3 heads in a row and then 3 tails in a row. Overall, that matches the P(tails) prediction that we gave in the beginning, but it will not match each time we flip a coin. Another way to look at it is that probability and ratio don't measure the same thing: probability is a predictor of what is to come and ratio is a reporter of what has happened. If it is a fair coin and fair tosses, eventually the ratio will match the probability.

Activity Two: In a deck of 52 playing cards (no jokers), how many cards are:

red? 26 black? 26 clubs? 13 diamonds? 13 hearts? 13 spades? 13

If you drew a card at random, what is P(red)? 26/52 = 1/2 P(heart)? 13/52 = 1/4 P(9 of hearts)? 1/52

Thoughts Before Beginning Statistics

Why Statistics?

Having a basic understanding of statistics is an important practical life skill. So many statistics are thrown about every day, often with only a fleeting reference to how the data were gathered or what the distribution of data looks like. Societally, we tend to take whatever the statistic-provider *says* the data show at face value when the more appropriate response might be to question how the questions were structured, how the data were gathered, and what story *the data* are trying to tell. We will not achieve the goal of creating critical consumers of data in the elementary years, but we can lay a strong and confident foundation!

Once upon a time, statistics was considered a specialized field of mathematics, taught sparingly at the elementary level. Now, the expectation is that children should have foundational concepts in statistics well in hand sometime during their sixth year, building on that foundation throughout the remainder of their academic career. This goes hand-in-hand with the belief that graphing should be integrated throughout a child's elementary years. Both of these seem to be academically and developmentally appropriate if properly taught. Doing so changes mathematics from being strictly computationally based activities to integrating number sense and the physical meaning of the work being done.

There are three concepts in Montessori philosophy that figure prominently in making concepts academically and developmentally appropriate:

- Ground each concept in the concrete
- Isolate the difficulty
- Teach each concept in relationship to the whole (including in relationship to prior learning)

<u>Ground in the Concrete</u>
Statistics are a man-made construct to describe an *assortment* of data.
Percents compare *a single data point* to the whole, much like fractions. As has been shown, this is easily demonstrated concretely, largely by using the 100 Board.
Ratios compare *two data points* at a time (at least as ratios have been taught thus far). It is wholly reasonable to show ratios with concrete objects like checkers.
The challenge arises when we wish to show a variety of qualities of an assortment of data points concretely. We will use concrete materials to introduce concepts whenever possible, relating them to graphs. The graphs then become the visual representation of the concrete rather than a mysterious plot on which we perform calculations.

<u>Isolate the Difficulty</u>
As children's ability to reason abstractly grows, Isolation of Difficulty looks a little different than it did in early childhood or even in lower elementary classrooms. Rather than striving always to have only one single concept in a given lesson, we *might* present several closely related concepts. We continue to isolate the difficulty within that lesson when we present each idea individually AND THEN intentionally compare and contrast them to distinguish one from the next, to produce clarity of each concept.

A word about Isolation of Difficulty applied to graphing: this important Montessori principle demands that children have experience with creating and interpreting graphs prior to applying that knowledge to statistics. While there are numerous lower elementary level standards related to graphing, it is wise to not presume that children have fully internalized what they need to be successful. Determine children's level of experience and expertise with graphs and remediate if needed before launching into statistics.

<u>Teach in Relationship to the Whole</u>

Three aspects of these lessons help establish the knowledge of how the new learning relates to the whole:

- Every lesson relates the new learning to prior learning in the introduction.
- Many problems are story based, relating statistics to everyday life.
- New concepts are compared and contrasted to other concepts presented in the lesson and to already-internalized concepts.

This provides much-needed context to what would otherwise be a very abstract and algorithm-driven mathematical subject.

The concept of teaching in relationship to the whole has particular relevance in applied statistics. Some curricular studies, like weather, lend themselves to data gathering and statistics. It is quite popular to have someone report on the weather daily. As long as the statistics are retained from day to day, they can make a wonderful database! Additionally, statistics often show up in current events. Unfortunately, there is rarely sufficient information to know how the data were gathered. But if a graph of any kind is provided, it can still be interesting to talk about the characteristics of the graph and what story it might be telling. Are the data tightly clustered, such that there really is little variation or does it have a wide range? (Also, for those who subscribe to USA Today, which has some kind of graph featured prominently virtually every day, remember that pie charts are statistically based information with data values expressed as a percent of the whole!)

In aggregate, through presenting new concepts as concretely as possible, intentionally managing the transition to abstract processes, and applying the concepts in real life or life-like circumstances, we can make statistics something as well understood and well internalized as other mathematical concepts. This foundation will enable children to better understand the ever more complex and abstract concepts that are to come in middle school and beyond.

Statistics
Statistics and Their Displays

Materials:

Checkers
Stamp Game
Graph paper and colored pens / pencils (hierarchical colors + black)
Graphs to show in the lessons (clean copies follow the lesson)

Direct Aim: to introduce the concept of statistics as different from a ratio or a pure number
To regard graphs as a means to depict statics

Indirect Aim: to create interest in statistics through collaboration
To lay the foundations of critically thinking about the ways data are presented

Applicable Standards:
CCSS.MATH.CONTENT.6.SP.A.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
CCSS.MATH.CONTENT.6.SP.B.4 (partial – italicized standard not addressed here) Display numerical data in plots on a number line, including dot plots, histograms, *and box plots.*

Prerequisites: knowledge and understanding of other means of expressing relative quantities, i.e. ratios and percents.
Success with or mastery of creating and interpreting pictographs, bar graphs, dot plots
Experience with Stem and Leaf charts and histograms is helpful, but not required

Presentation:

1. Place 4 red checkers and 2 black checkers on the rug/table. Characterize them as 4 red circles and 2 black circles. Ask a volunteer to exercise a little effortful retrieval to provide the following:
   - The ratio of red circles to black circles. <4:2 or 2:1>
   - The ratio of black circles to red circles. <2:4 or 1:2>
   - The fraction of circles that are red. <4/6 or 2/3>

   Ask a volunteer to explain why a fraction is a ratio but not all ratios are fractions. <Fractions express the relationship between quantities like all ratios do. It is a specialized ratio, because the second number always represents the number that is associated with a whole or the total.>

2. Add 6 red squares, 7 green squares, and 1 blue square from the stamp game. (Place stamps with the numerical value face down.) Ask a volunteer to provide the following:
   - The ratio of red circles to red squares. <4:6 or 2:3>
   - The fraction of pieces that are red. <10/20 or 1/2>
   - The ratio of circles to squares. <6:14 or 3:7>
   - The ratio of squares to circles. <14:6 or 7:3>
   - The ratio of red to not-red. <10:10 or 1:1>

3. Throughout history, humans have had the need to express how many there are of something. In some ways, the entire field of mathematics is all about "how many". We have lots of different ways of expressing absolute and relative quantities.
   Numbers allow us to answer the question, "How many are there?" in an absolute sense. We count and report our findings or we perform an operation and report the answer.
   Ratios allow us to answer the question, "How much of there is *this* compared to *that*?"
   Fractions and percents allow us to answer the question, "How many are there compared to the whole?"

   Sometimes when we ask a question, we want to know all the possible responses and how many there are of each response. For example, we could ask, "What shapes and colors are out on the table or mat and how many are there of each?"
   - We could answer that question with *absolute numbers*, but counting each and making a list: 4 red circles and 2 black circles and 6 red squares and 7 green squares and 1 blue square.
   - We could try to answer that with *a ratio* that tells relative quantities: the ratio of red circles to black squares to red squares to green squares to blue squares is 4:2:6:7:1. (That is awkward!) Or by trying to express each ratio pair that we can think of, (That would take a long time, and might not really help us understand things any better!
   - We could answer that using *percentages*: 20% red circles, 10% black circles, 30% red squares, 35% green squares and 5% blue squares.

   If we wanted to know the specific numbers of each shape/color combination <u>and see how they all relate to one another</u>, we would use *statistics*.

4. We display statistics in a table or graphically. Often, the left column describes a quality or value, while the right column displays a quantity or frequency – how many match the quality described in the left column.

   Start the table to the right and ask the children to complete it. < Results shown in italics.>

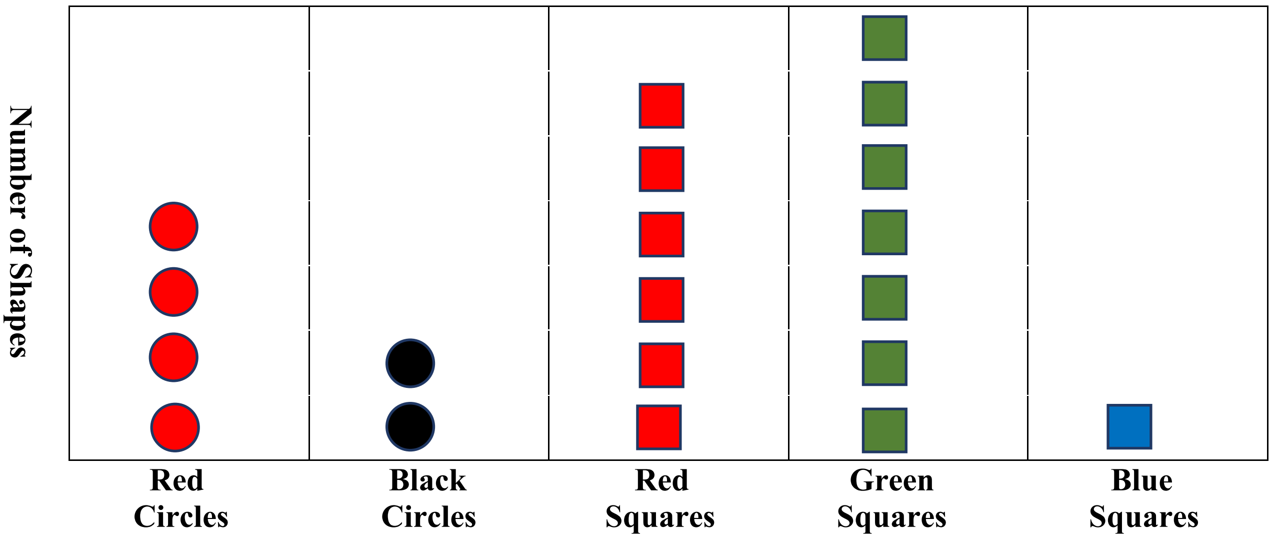
   Recall that the question was, "What shapes and colors are out on the table or mat and how many are there of each?"
   Solicit opinions about the information provided by the chart compared to the list of numbers, the giant ratio, and the list of percents.

Quality or Value	**Quantity that Match or Frequency**
red circle	4
black circle	*2*
red square	*6*
green square	*7*
blue square	*1*

5. What type of graph would be good to display these data absolutely?
   <There are a few good options: bar graph, pictograph and dot plot.>

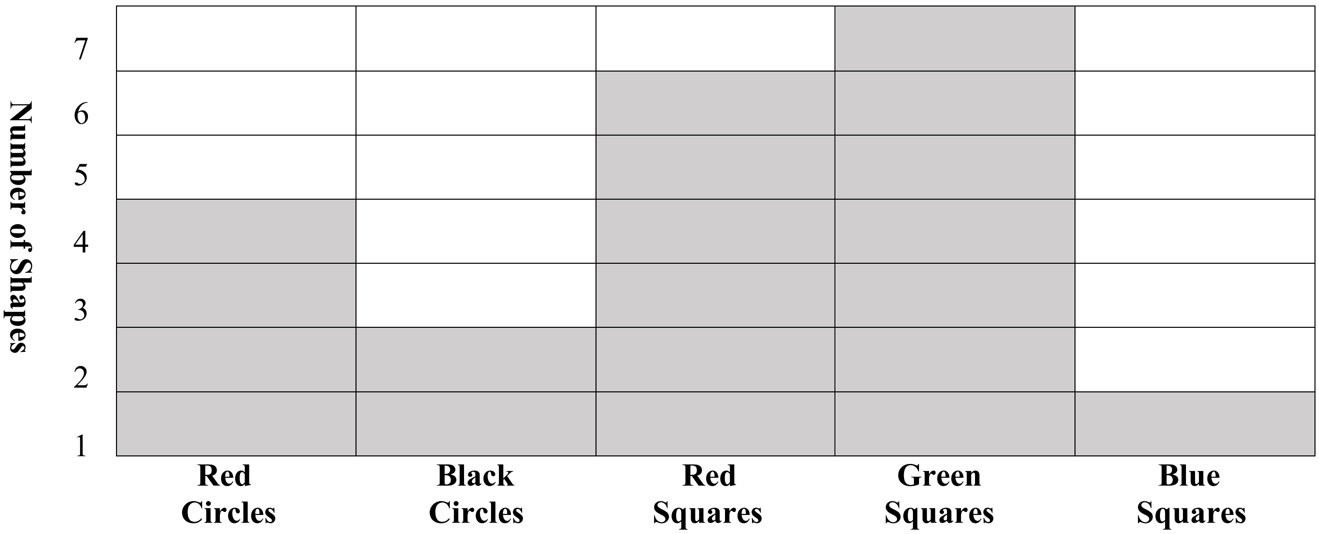
6. If children have sufficient experience with these three graphing techniques, ask them to split up into 3 groups (pairs? individuals?) with each group/individual creating one of the three types of plots. Results should look something like those on the following pages.

Pictograph



Note: this graph was constructed building up from the bottom horizontal axis. It is equally correct to build to the right from the left vertical axis. If the children created a vertical display like this one, arrange the objects by color and shape into rows to imitate what a horizontal pictograph would look like, just to show alternatives. If they created a horizontal display, arrange the pieces into columns (like shown above) to show the vertical orientation.
Explore the idea that sometimes pictographs have a scale. For example, if there was a key that indicated that every icon represented 2 shapes of that type and color, this graph would be telling us that there are 8 red circles, 4 black circles, 12 red squares, 14 green squares, and 2 blue squares on the table or rug
It is not necessary to have an icon that looks like the object being represented since the bottom axis is labeled with the shape type and color. It could be that all shapes are represented by a single icon like a check mark or a snowflake, In this case, since the icon being used is literal (a red circular checker is represented by a red circle), it is not necessary to have a key, but it would not be incorrect to have one.

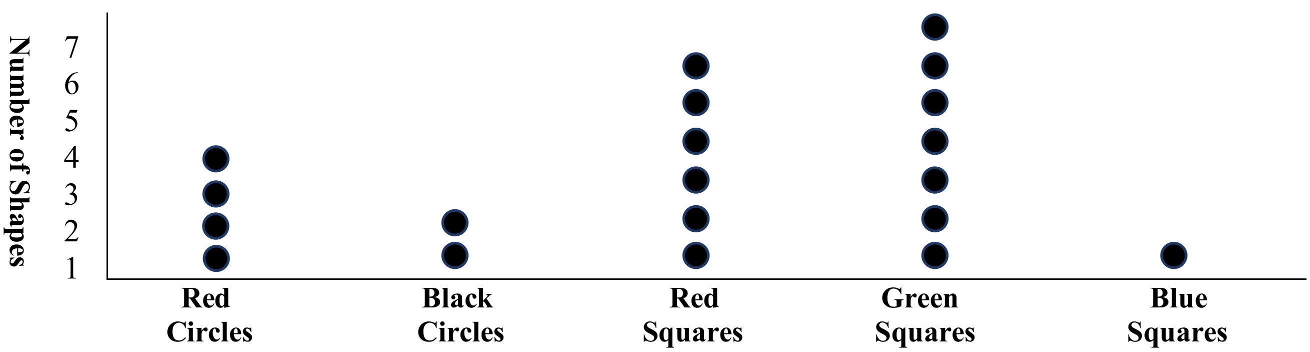
Bar Graph



Note: as was true with the pictograph, this type of graph is equally correct with a vertical orientation (as above) or with a horizontal orientation. FYI: bar graphs with a vertical orientation like these are sometimes referred to as column graphs.

This graph was constructed with the values (shapes and colors) graphed in the sequence in which the data were provided. It is equally correct to sequence the shapes from most frequently occurring to least or from least to most.

Dot Plot



This is a third way to display the data. As was the case with the bar graph, this graph was constructed with the shapes graphed in the sequence in which the data were provided. It is equally correct to sequence the shapes from most frequently occurring to least or from least to most.

7. Compare the results. Accept all well-reasoned observations.
   Recall that the question was, “What shapes and colors are out on the table or mat and how many are there of each?”
   Compare how the three charts depict the answer to that question. Solicit opinions about the quality of information provided by the charts compared to the table of numbers.

8. Remove the checkers and stamps. Explain that gathering statistics begins with a question. Statistical questions expect multiple different responses. If we ask one person a factual question like, "What is your shoe size?" we would get a number, a numerical answer, but it would be a single number with no variability. If we asked the whole class or the whole school or the whole state about shoe sizes and compared the results, we would get lots of responses, and there would be variability in the answers.

   Imagine that we asked everyone in our class how far they lived from school. We would record how far each person travels to get to school. When we look at all the numbers in a table or on a graph, we can see how many people live closer and how many live farther. That is what statistics does – it looks at all the possible values and their distribution.

9. Ask the children if each of these is a statistical or a numerical question:
   - How many siblings does Glenn have? <no>
   - What size families do children in our schools come from? <yes>
   - What movies everyone in America attended last weekend? <yes>
   - What movie I last saw? <no>

   Continue until children show confidence in the concept. For extra challenge, ask a volunteer to answer a question like, "What is your favorite color?" and invite a volunteer to turn that into a statistical question like, "What are the most and least common favorite colors of everyone in our class?"

10. The statistics we graphed earlier (the number of each of 5 different shape/color pairs) is pretty simple: there were only 5 values (shapes and colors) and only 20 individual pieces (quantity/frequency). That lends itself well to charts and the types of graphs where each answer becomes a character on the graph (a pictograph, bar chart, or dot plot). Statistics can get more complex when there are more response types (values), more responses (frequencies), or both.

    If children seem like this is new news, please end the lesson here and provide practice with pictographs, bar charts, and dot plots. If they have had sufficient experience with these graphs are indicating relatively high levels of confidence, continue with the lesson. *Any commercial product that asks children to create or interpret pictographs, bar charts, dot plots, stem and leaf plots or histograms would be suitable.*

11. Propose the following new story:
    Jess is planning a <u>big</u> family reunion. To have activities that everyone will enjoy, it is important to know the age of each attendee. Jess writes the ages of all the children from youngest to oldest:
    1, 1, 2, 2, 3, 4, 4, 4, 4, 5, 8, 10, 10, 10, 11, 11, 13, 17, 19, 25, 25, 27, 27, 28, 28, 29, 30, 31, 31, 31, 32, 32, 32, 34, 35, 40, 44, 48, 49, 52, 56, 57, 57, 58, 63, 66.

    Wow – what a mess! That is hard to look at and tell what is going on.

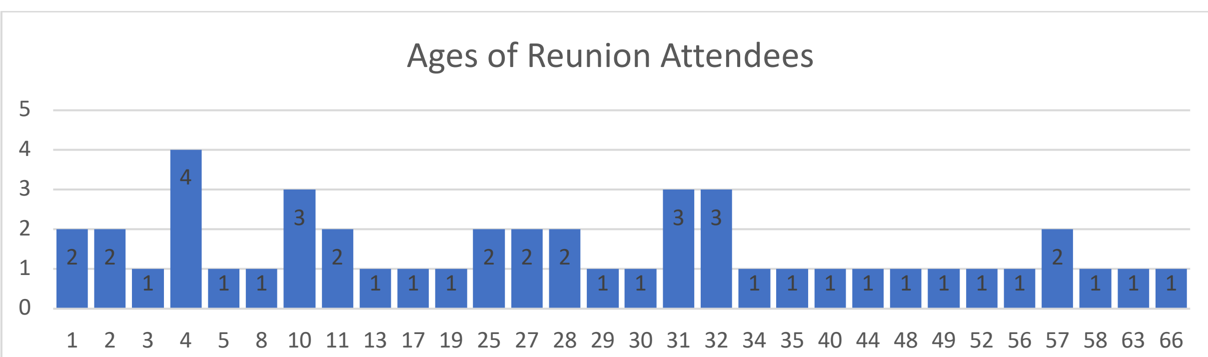
In an attempt to make the data "speak", Jess puts it into a traditional 2-column chart, suitable for a bar graph or dot plot.

Note: full-scale, clean copy of this table follows the lesson

Age	Number of Attendees
1	2
2	2
3	1
4	4
5	1
8	1
10	3
11	2
13	1
17	1
19	1
25	2
27	2
28	2
29	1
30	1
31	3
32	3
34	1
35	1
40	1
44	1
48	1
49	1
52	1
56	1
57	2
58	1
63	1
66	1

When the chart is done, she despairs – it is not much better! There are some individuals who are of a common age, but not many.

Jess could do a bar graph or a dot plot to show the distribution of ages, but there would be 66 columns! It would not tell her much. Show the following graph (clean copy follows the lesson). Ask what the data show and what value that information has for Jess.



What should Jess do? *Discuss what Jess really needs to know. Does she need to know the exact age of each attendee? <No, she needs to know the number of people in age <u>groups</u>.>*

Stem and Leaf Plot – Age of Attendees
Jess knows her statistics tools! She begins with a stem and leaf chart. She decides to group attendees into 10-year age ranges on a stem-and-leaf chart, where the first digit of the age (the tens) is split out from the units' digit.
Demonstrate creating a stem and leaf plot like the one on the right.

Stem (Tens' digit)	**Leaf** Ones' digits
0	1, 1, 2, 2, 3, 4, 4, 4, 4, 5, 8
1	0, 0, 0, 1, 1, 3,7, 9
2	5, 5, 7, 7, 8, 8, 9
3	0, 1, 1, 1, 2, 2, 2, 4, 5
4	0, 4, 8, 9
5	2, 6, 7, 7, 8
6	3, 6

Key: 6|3 = 63

Point out that both charts are showing identical data. It is just displayed differently. With the stem-and-leaf chart, the data are grouped.

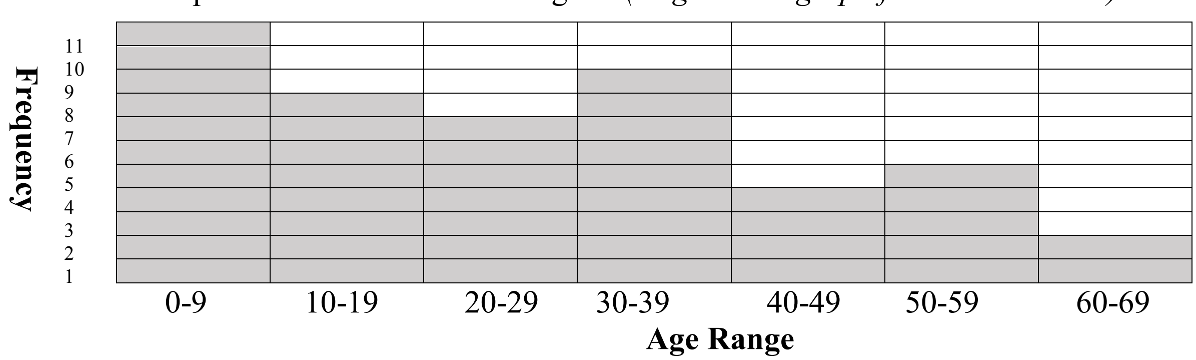
Discuss the differences in the displays and in what a person can glean from the display. <Accept all well-reasoned responses. If needed, lead children in more in-depth observations like the notion that there are actually very few people over the age of 40 compared to the rest of the group, or to the observation that there are a LOT of kids under the age of 10.>

Continue the story…
Since Jess knows a lot about tools for statistics, she knows that she can make a histogram of these data. (If needed, explain that a histogram is very similar to a bar graph, but each bar represents a *range* of values rather than a single value.) First, from the stem and leaf plot, she makes a 2-column chart showing how many attendees there are in each age group.

In the left column is the categories – the age ranges into which she will sort the data. The right column has the variables – the number of attendees that fit each category.

Category Age range	**Variable** # of attendees
0-9	11
10 - 19	8
20-29	7
30 - 39	9
40 – 49	4
50 - 59	5
60 - 69	2

Then she plots these values on a histogram *(larger scale graph follows the lesson)*:



12. Summarize the experiences with statistical charts and graphs from the lesson.
    - Traditional 2-column charts that show the number of responses for each possible response type lead naturally to pictographs or bar graphs or dot plots. These charts and graphs are great for statistics that have only a few possible responses.
    - A Stem-and-Leaf chart shows groups of responses. It is great for statistics that have lots of possible responses that can logically be grouped. It leads naturally to a histogram.

13. Ask the children to practice a little with these concepts using the provided follow-up or one of your own choosing. Explain that after everyone has concepts well in hand, each student will get to design and carry out a statistical question! They will get to survey people in the class about shoe size or number of pets at home OR to set up an observation, like learning the most popular color for cars in car-pool at school – anything that they want to know about statistically!

Follow-up

The first part of the follow-up activity (which follows several pages of displays for the lesson) deepens children's understanding of the various charts and plots/graphs discussed in this lesson. It helps them see graphs as a statistical tool. If children were well grounded in most or all of the graph types prior to this lesson, the amount of practice in the follow-up may well be sufficient to accomplish the direct aims of the lesson. If children have little prior graphing experience, more practice will be needed. *Note – there are 4 questions on 4 separate pages due to the space needed for graphs and charts.*

Once the children have completed however much practice they need to become facile with these graphs, regather to discuss the project, (this might well be another day).

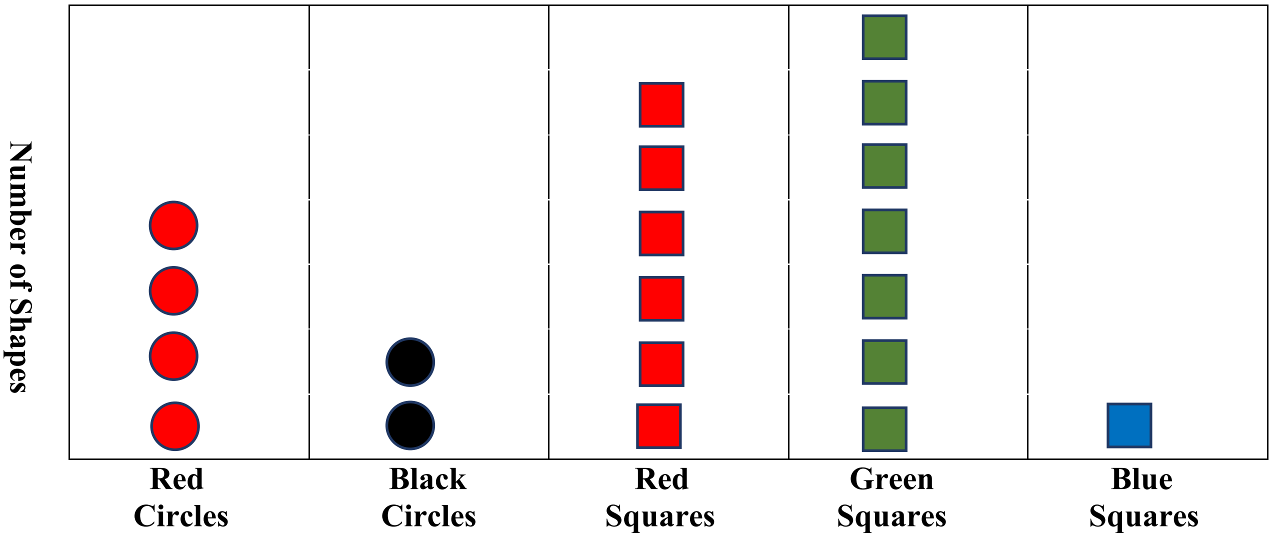
The project invites children to gather data in response to a statistical question of their own design and then graph the results. Spending some time brainstorming questions and the types of graphs that they might suggest will give children a good head start! Let the instructions for the project guide the brainstorming session.

Some of the most important guidance for personal surveys, when the time comes:

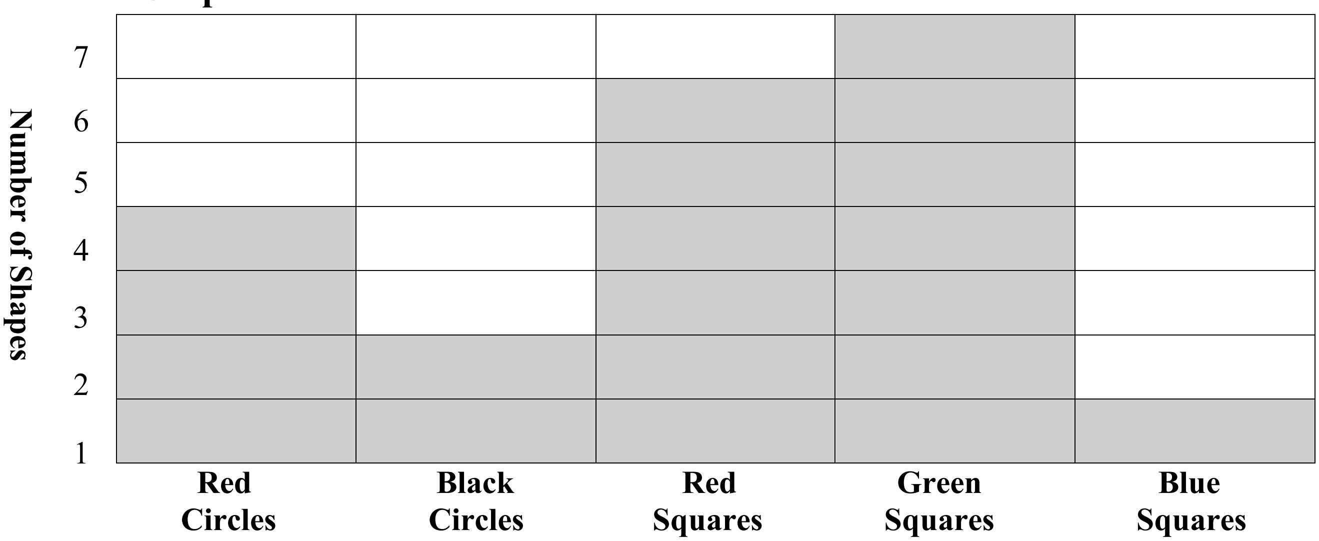
- Questions that have 4-7 possible responses (values) will feed nicely to 2-column chart to capture the data and will graph nicely on a pictograph, a dot plot or a bar graph. Questions that have lots and lots of possible answers that will need to be grouped lend themselves to a stem and leaf plot and a histogram.
- The least time-efficient method of gathering data is to walk around with a clipboard, asking everyone the same question, trying to figure out who still needs to respond. Brainstorm ways to minimize that, such as:
    - announcing the polling question to the whole class right before recess (with permission) and then standing at the door with a clipboard to record each response as children leave the room OR
    - writing the question on paper and giving a copy to each person to complete and return by the end of the day, like voting.

Graphs for Introductory Lesson

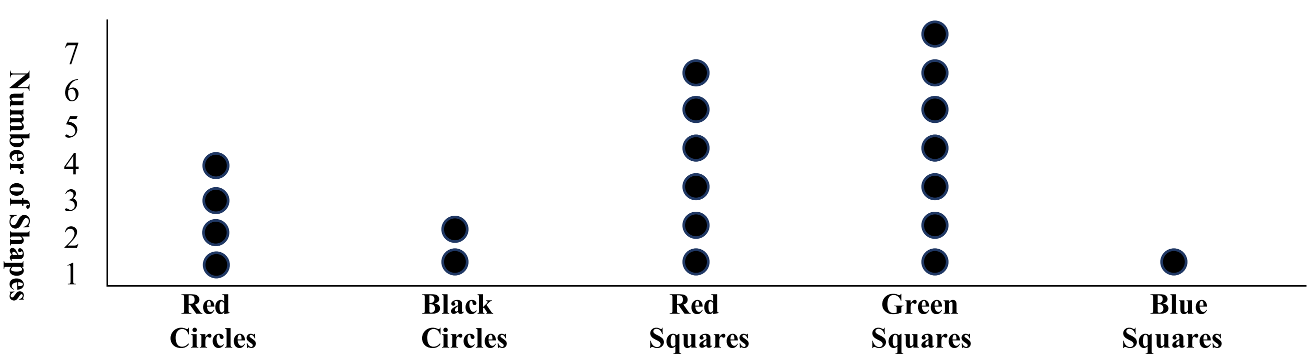
Pictograph



Bar Graph



Dot Plot



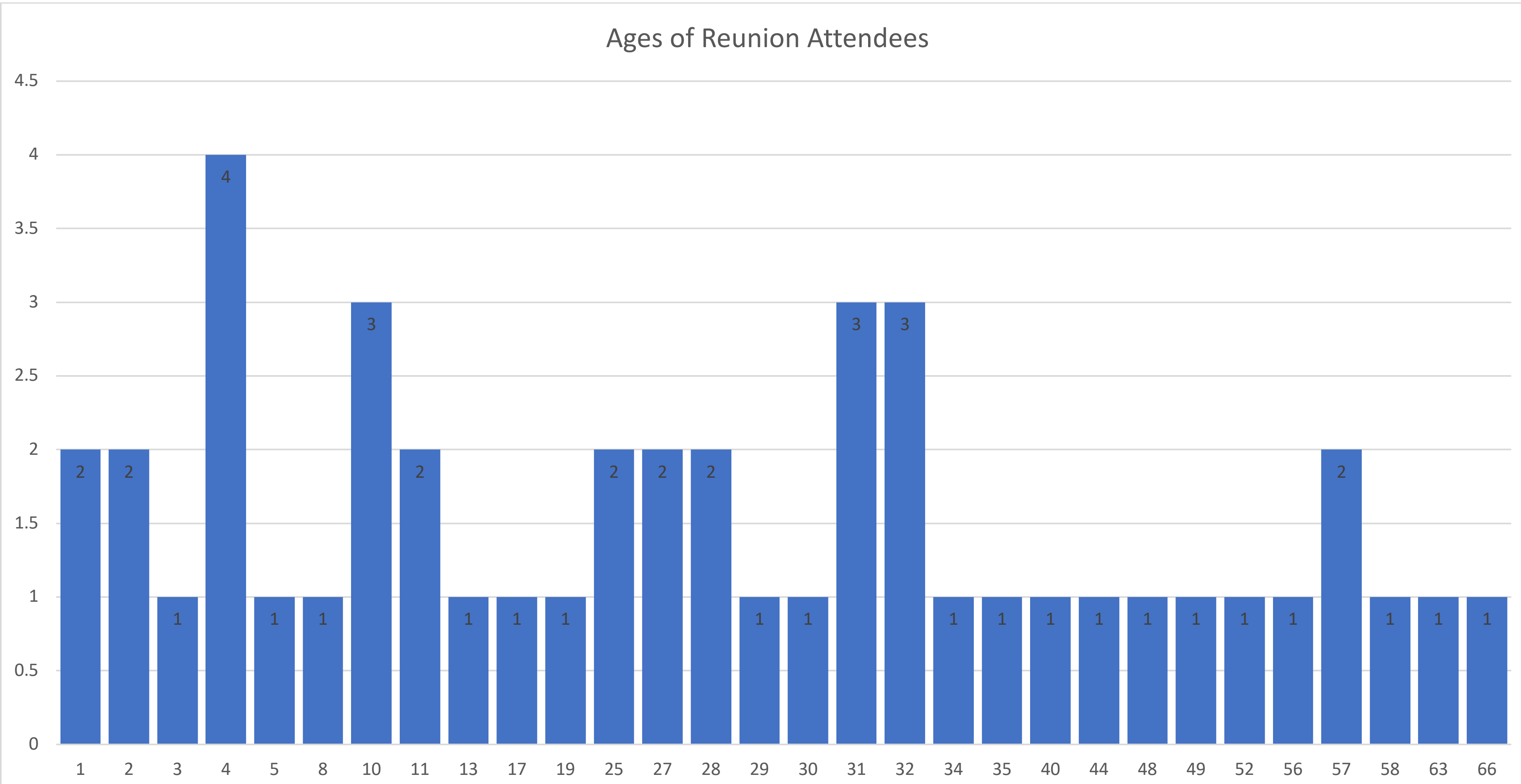
2-Column Chart
Reunion Attendees

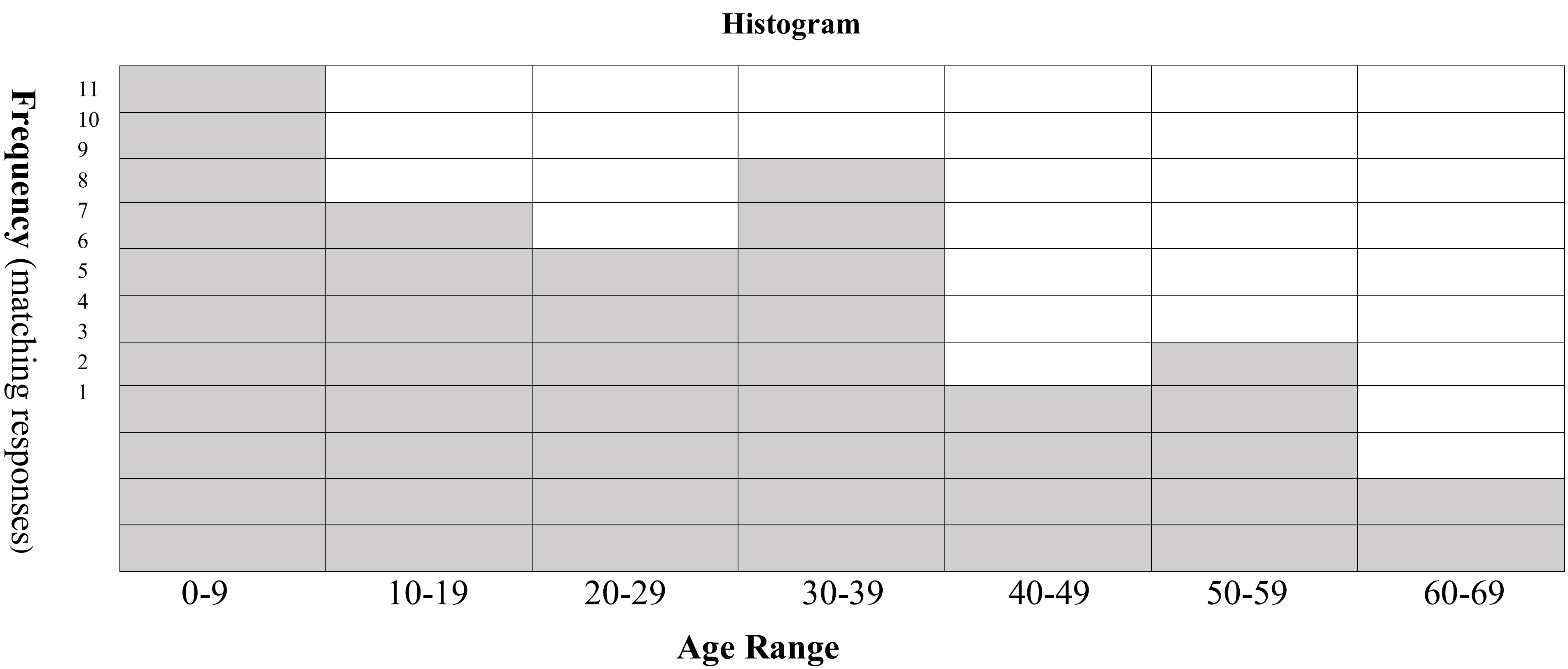
Age	Number of Attendees
1	2
2	2
3	1
4	4
5	1
8	1
10	3
11	2
13	1
17	1
19	1
25	2
27	2
28	2
29	1
30	1
31	3
32	3
34	1
35	1
40	1
44	1
48	1
49	1
52	1
56	1
57	2
58	1
63	1
66	1

Stem and Leaf Plot
Reunion Attendees

Stem (Tens' digit)	**Leaf** Ones' digits
0	1, 1, 2, 2, 3, 4, 4, 4, 4, 5, 8
1	0, 0, 0, 1, 1, 3,7, 9
2	5, 5, 7, 7, 8, 8, 9
3	0, 1, 1, 1, 2, 2, 2, 4, 5
4	0, 4, 8, 9
5	2, 6, 7, 7, 8
6	3, 6

Bar Graph


Ages of Reunion Attendees
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1 2 3 4 5 8 10 11 13 17 19 25 27 28 29 30 31 32 34 35 40 44 48 49 52 56 57 58 63 66
2 2 1 4 1 1 3 2 1 1 1 2 2 2 1 1 3 3 1 1 1 1 1 1 1 1 2 1 1 1


Histogram
Frequency (matching responses)
11
10
9
8
7
6
5
4
3
2
1
0-9
10-19
20-29
30-39
40-49
50-59
60-69
Age Range

Statistics and Their Displays (1)

Pizza π

The Pizza π Restaurant has been open for 7 years. They have tracked the total number of pizzas sold each year since the store opened. The results are shown here, rounded to the nearest 500 pizzas.

Year	Pizzas Served
1	1,500
2	3,500
3	5,000
4	7,500
5	7,500
6	5,500
7	12,000

Create a pictograph that shows these data. Use a scale of 1 icon symbol = 1000 pizzas.
NOTE: Keep your icon simple - you will sometimes use half-symbols in your pictograph.

Show the key on your graph. You may wish to use graph paper to create your graph and glue or staple the graph to this worksheet.

Answer the following questions.

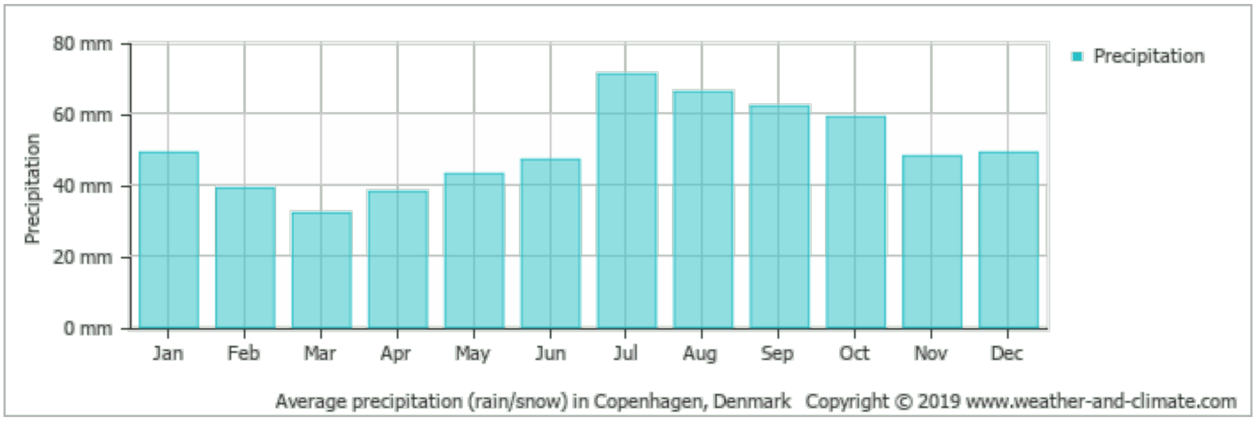
1. In what year was there no growth in pizza sales from the previous year?

2. One year, the owners of Pizza π decided that they had to expand the restaurant. The restaurant closed for almost 3 months due to the construction. They nearly doubled the size of the restaurant. What year do the data suggest was the year of construction?

3. The year following the construction and renovation had record-breaking sales. How many more pizzas did Pizza π that year than in the year prior to the construction?

Statistics and Their Displays (2)

Copenhagen's Climate
Copenhagen is about 55ºN latitude – roughly on a parallel with Russia, Kazakhstan, Southeast Alaska, and Quebec. It is surrounded by ocean, which keeps its temperatures moderate and humidity high. Here is a graph of the average precipitation on a month-to-month basis.



Create a 2-column chart from this graph, indicating the month of the year and the millimeters of precipitation they typically get. Then answer the following questions:

Month	**Precipitation**
January	
February	
March	
April	
May	
June	
July	
August	
September	
October	
November	
December	

1. About how much more precipitation does the wettest month receive than the driest month? have the most and the least precipitation on average?

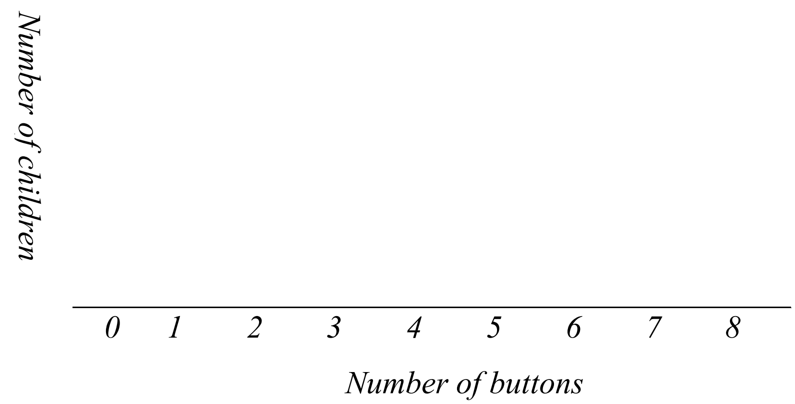
2. Which months have at least twice as much precipitation as the driest month?

3. How would you define the rainy season, based on this graph?

Statistics and Their Displays (3)

Button, button, who's got a button?
A teacher who works with children ages 3-6 has been giving a lot of lessons on the button frame. He began to think about the children's clothing. So many have pants with elastic waists and t-shirts or sweaters. He started to question if children had ample opportunities to practice buttoning when they were not at school. He decided to gather some data. He stood by the door to go to the playground and did a quick count of how many buttons were on each child's outfit. Here are their results: 0, 0, 4, 6, 4 , 0, 2, 1, 8, 0, 5, 4, 2, 3, 1, 0, 8, 3, 6, 4, 7, 6, 0, 5, 2. Create a *2-column table* and a *dot plot* to show their results. The table and dot plot have been started for you here or you can create one on graph paper:

Number of buttons	**Number of children**
0	6
1	



1. How would this graph be different if it were a bar graph?

2. What would you say to the teacher about how many opportunities children in their class have to practice buttoning on their own clothing?

Statistics and Their Displays (4)

Caffeine Crisis

The Starbucks in Los Alamos, New Mexico is a smaller stand-alone store. The daily printout shows the number of customers per hour on one day in May. The manager, Caitlyn, wants to see what the distribution looks like – when there are coffee rush-hours. She decides that this table has too many data points to look at – it is hard to get any real meaning from the data. She decides to make stem and leaf plot and a histogram. Please help Caitlyn out! (It has been started for you)

Time	# Customers/hr.
5:00-6:00	20
6:00-7:00	42
7:00-8:00	87
8:00-9:00	60
9:00-10:00	52
10:00-11:00	36
11:00-12:00	44
12:00-1:00	46
1:00-2:00	41
2:00-3:00	32
3:00-4:00	59
4:00-5:00	38
5:00-6:00	17
6:00-7:00	8
7:00-8:00	1

Stem and Leaf Plot
Number of Customers per Hour
Starbucks Coffee, Los Alamos, New Mexico
May 27

Stem (Tens' digit)	**Leaf** (Ones' digits)
0	8,1

Then she plots these values on a histogram. The first one is done for you:

Caitlyn learned these things:
There were ____ times during the day when there are more than 40 customers per hour.
There were ____ times in the day with more than 70 customers per hour.

Caitlin wanted to see when these peak times happened during the day.
Go back to the original 2-column chart and put a star by any hours that have more than 40 customers per hour. Put 2 stars by any hours with more than 60 customers in an hour.
What staffing recommendations would you make to Caitlyn?

Control of Error for Statistics and Their Displays (1)

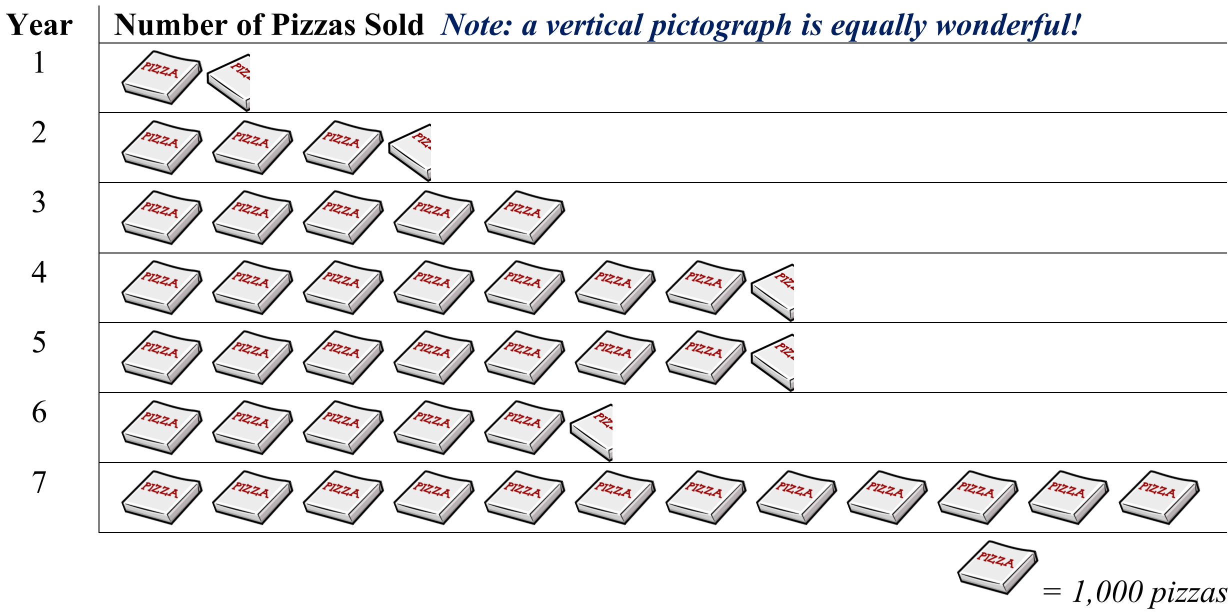
Pizza π

The Pizza π Restaurant has been open for 7 years. They have tracked the total number of pizzas sold each year since the store opened. The results are shown here, rounded to the nearest 500 pizzas.

Year	Pizzas Served
1	1,500
2	3,500
3	5,000
4	7,500
5	7,500
6	5,500
7	12,000

Create a pictograph that shows these data. Use a scale of 1 icon symbol = 1000 pizzas.
NOTE: Keep your icon simple - you will sometimes use half-symbols in your pictograph.

Show the key on your graph. You may wish to use graph paper to create your graph and glue or staple the graph to this worksheet.



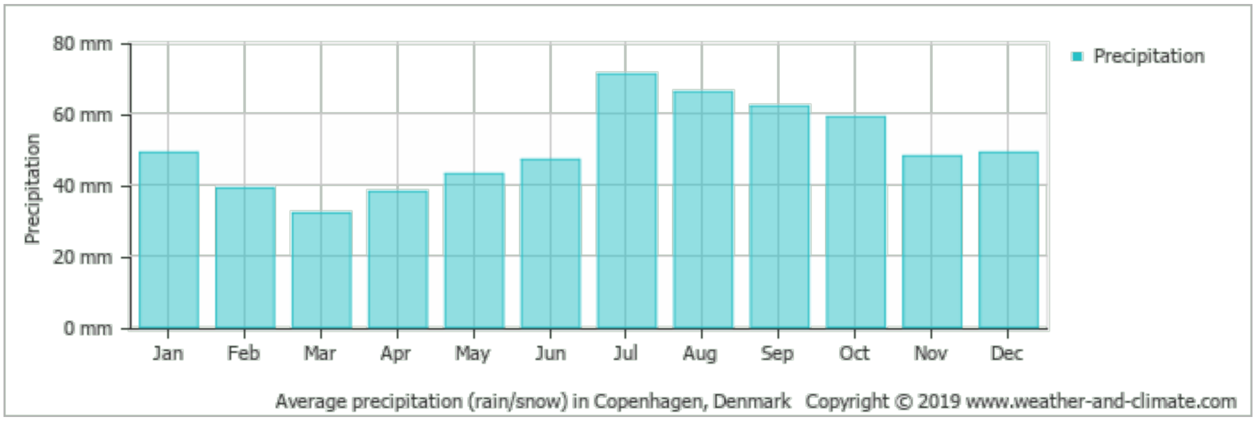
1. In what year was there no growth in pizza sales from the previous year? *Year 5 – sales were the same as year 4.*

2. One year, the owners of Pizza π decided that they had to expand the restaurant. The restaurant closed for almost 3 months due to the construction. They nearly doubled the size of the restaurant. What year do the data suggest was the year of construction?
Year 6, when sales dropped by 2000 pizzas compared to the previous year. Notice how much sales went up following the expansion!

3. The year following the construction and renovation had record-breaking sales. How many more pizzas did Pizza π that year than in the year prior to the construction? *In year 7, Pizza Pi sold 12,000 pizzas. The year before the construction they sold 7,500 – an increase of 4500 pizzas!*

Control of Error for Statistics and Their Displays (2)

Copenhagen's Climate
Copenhagen is about 55ºN latitude – roughly on a parallel with Russia, Kazakhstan, Southeast Alaska, and Quebec. It is surrounded by ocean, which keeps its temperatures moderate and humidity high. Here is a graph of the average precipitation on a month-to-month basis.



Create a 2-column chart from this graph, indicating the month of the year and the millimeters of precipitation they typically get. Then answer the following questions:

Month	**Precipitation** *Answers will vary a bit*
January	*50*
February	*40*
March	*30*
April	*35*
May	*42*
June	*45*
July	*70*
August	*65*
September	*62*
October	*60*
November	*50*
December	*55*

1. How much more precipitation does the wettest month receive than the driest month? have the most and the least precipitation on average? *At 70 mm., July is the wettest and March, at 30 mm, is the driest. The difference is 40mm.*

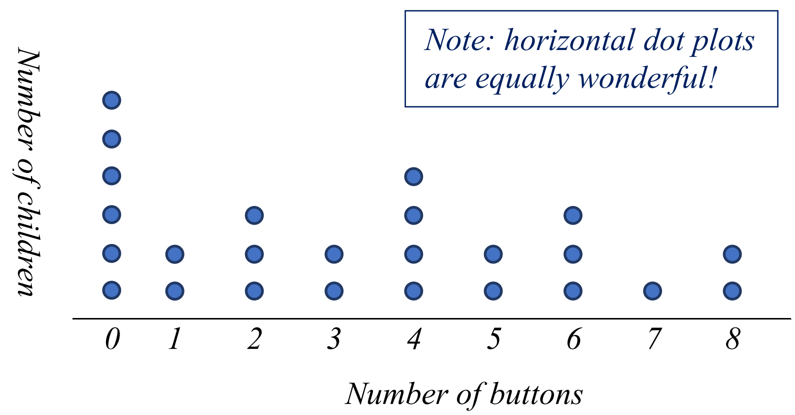
2. Which months have at least twice as much precipitation as the driest month?
July, August, September, and October have 60 mm or more precipitation per month.

3. How would you define the rainy season, based on this graph?
The rainy season begins abruptly in July and goes through the end of the year and into the following January.

Control of Error for Statistics and Their Displays (3)

Button, button, who's got a button?
A teacher who works with children ages 3-6 has been giving a lot of lessons on the button frame. He began to think about the children's clothing. So many have pants with elastic waists and t-shirts or sweaters. He started to question if children had ample opportunities to practice buttoning when they were not at school. He decided to gather some data. He stood by the door to go to the playground and did a quick count of how many buttons were on each child's outfit (including outerwear). Here are their results: 0, 0, 4, 6, 4 , 0, 2, 1, 8, 0, 5, 4, 2, 3, 1, 0, 8, 3, 6, 4, 7, 6, 0, 5, 2. Create a *2-column table* and a *dot plot* to show their results:

Number of buttons	Number of children
0	6
1	*2*
2	*3*
3	*2*
4	*4*
5	*2*
6	*3*
7	*1*
8	*2*



1. How would this graph be different if it were a bar graph?
   It would not be very different. Instead of individual dots, there would be a bar for each column of dots. The only difference is that the dots can be counted; there is a 1:1 correspondence between each dot and a child in the class, whereas the bar shows the number of children with a particular number of buttons by its height

2. What would you say to the teacher about how many opportunities children in their class have to practice buttoning on their own clothing?
   Answers will vary but may include agreeing that many children do not have many buttons on their clothing. In fact, 13 children have fewer than 4 buttons on their outfit on the day of the survey.

Control of Error for Statistics and Their Displays (4)

Caffeine Crisis

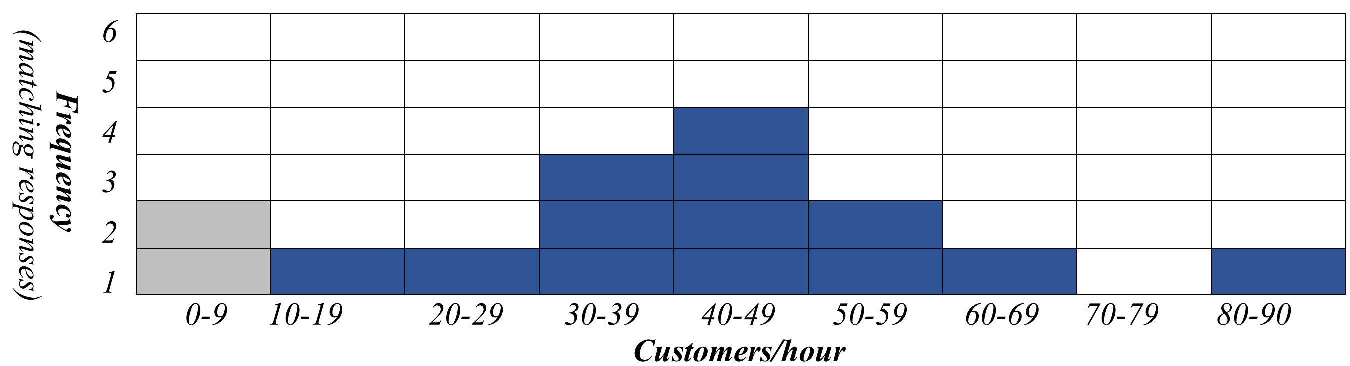
The Starbucks in Los Alamos, New Mexico is a smaller stand-alone store. The daily printout shows the number of customers per hour on one day in May. The manager, Caitlyn, wants to see what the distribution looks like – when there are coffee rush-hours. She decides that this table has too many data points to look at – it is hard to get any real meaning. She decides to make stem and leaf plot and a histogram. Please help Caitlyn out! (It has been started for you)

Time	# Customers/hr.
5:00-6:00	20
6:00-7:00	★ 42
7:00-8:00	★★ 87
8:00-9:00	★★ 60
9:00-10:00	★ 52
10:00-11:00	36
11:00-12:00	★ 44
12:00-1:00	★ 46
1:00-2:00	★ 41
2:00-3:00	32
3:00-4:00	★ 59
4:00-5:00	38
5:00-6:00	17
6:00-7:00	8
7:00-8:00	1

Stem and Leaf Plot – Number of Customers per Hour
Starbucks Coffee, Los Alamos, New Mexico
May 27

Stem (Tens' digit)	**Leaf** Ones' digits
0	8, 1
1	*7*
2	*0*
3	*6, 3, 8*
4	*2, 4, 6, 1*
5	*2,9*
6	*0*
7	
8	*7*

Then she plots these values on a histogram:



Caitlyn learned these things:
There were _8_ times during the day when there are more than 40 customers per hour.
There was _1_ time in the day with more than 70 customers per hour.

Caitlin wanted to see when these peak times happened during the day.
Go back to the original 2-column chart and put a star by any hours that have more than 40 customers per hour. Put 2 stars by any hours with more than 60 customers in an hour.
What staffing recommendations would you make to Caitlyn?
Answers will vary but will likely include heavy staffing from 7AM – 9 or 10AM and very light staffing from 5AM – 7AM and from 4PM – close, or maybe even recommend closing at 7PM.