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A Few Thoughts Before Beginning Numeration Systems
Roman Numerals and Alternate Base Systems

Thoughts on Roman Numerals
Why, in an already crowded curriculum, would a guide choose to include the study of Roman Numerals?

The ability to read Roman Numerals is a dying art. One might justifiably say that it is not the practical life knowledge that it once was. The truth is that there is little or no outside accountability for children to have extensive knowledge in this area. None-the-less, Montessorians continue to teach Roman Numerals because they are part of the extended story of the Fifth Great Lesson. In addition to creating a dot on a timeline of the advancement of numeracy, they contribute to the overall story of two quite dichotomous cultures: Ancient Greece and Rome. Greek culture was characterized by the proliferation of ideas relating to mathematics, geometry, astronomy, physics and the arts simply for joy of inquiry and knowledge – they valued mathematics as a key to unlocking untold mysteries of the Universe. This is in sharp contrast to the more dogmatic, pragmatic Roman culture, which used mathematics primarily as a tool for counting and accounting.

In addition, some Montessori and traditional pedagogies continue to teach Roman Numerals to enable children to read a clock with Roman Numerals or to continue to have access to historical information, from the dates on buildings or films to the history of the American football (Super Bowl L). In some ways, these arguments in favor of teaching Roman Numerals are analogous to reasons some give for teaching cursive writing.

These reasons alone do not create a compelling argument for giving them a place of honor in our curriculum. But there are real educational benefits to teaching Roman Numerals to elementary-age children:

- Math is all about patterns. The best mathematicians are pattern seekers. It only takes a few lessons to provide children with the basics of the Roman numeration system. This content encourages flexible thinking and trains the brain to detect a different kind of pattern – one with subtractive notation (IV)!
- Studying Roman Numerals is more symbolic and less computational than many other mathematical studies. As such, there are some children who see themselves as being “bad at math” who can find success with Roman Numerals. And some children who are skilled at cranking out numerical solutions to problems will need to pause and rethink the way they look at math. This benefits both kinds of learners, and those who find joy or challenge can use these two lessons as a launching point for further independent work.
- By studying Roman Numerals, which do not use positional notation (place value), children come to a deeper appreciation for what place value does in our numeration system. When we look at something that we know from a different perspective, we strengthen our semantic understanding of the subject.
Thoughts on Alternate Base Systems

Many of the same arguments for teaching Roman Numerals speak in favor of offering at least some exposure to non-decimal bases. It significantly enhances children’s semantic understanding of Base 10 – arguably even more than a study in Roman Numerals - and helps children refine their pattern-detecting skills. In addition to grounding children in the decimal hierarchy of numbers, it changes the way that they look at scientific and exponential notation.

In a sense, it is a bit like the old parable about the blind men who had their first encounter with an elephant. One, feeling the elephant’s side, says that the elephant is like a wall. Another, feeling the trunk, says that the elephant is like a snake. A third, feeling the tusk, says that the elephant is like a spear. By exposing children to alternate base systems, we effectively remove the blinders that they have worn since first exploring the decimal system with Golden Beads. They see how the system functions from a totally different perspective. They see the whole elephant – or at least a good portion of it!

This study requires more number sense than Roman Numerals, but not a great deal more computation. As long as children have some facility with math facts and the ability to think flexibly, they can participate in these lessons with great success. The only children who might not benefit from these lessons are those who are so tentative with number sense that knowing that there is more than one system of numeration would shake their confidence. These children can happily live into adulthood without the knowledge of other bases.

What is offered here is merely introductory lessons – children take it as far as they want. Some years, children in my class wanted to pursue addition and subtraction in binary and other bases. One year, when satisfied that they had mastered those concepts, they explored multiplication a bit, concluding that without significant exploration, they could best understand multiplication in other bases as repeated addition – a decision that matched my own conclusions!

In essence, Roman Numerals explores the merits of positional notation and Alternate Base Systems plumbs the depths of place value. Children love the code-breaking nature of these works, and they can have great success with these ideas almost irrespective of their prowess in other areas of mathematics.
Alternate Numeration Systems

Alternate Numeration Systems: Roman Numerals

Roman Numerals I

Materials: Tickets with images of sheaves of wheat (follows)
Tickets bearing various Arabic and Roman numerals (follow)
Small white board and marker
OPTIONAL: Maps showing the extent of Ancient Greek Civilization and of the Roman Empire, and other illustrations to support the story of the historical context for the evolution of Roman Numerals.
OPTIONAL: Displays comparing aspects of Greek and Roman civilization (links for lesson discussion or to jump-start student research follow within the lesson)

Direct Aim: to understand the Roman system of numeration sufficiently well to be able to read an analog clock and interpret Roman numerals as used in ordinal numeration (i.e. Super Bowl LVI).

Extended Direct Aim: to understand the Roman system of numeration sufficiently well to be able to read larger numbers such as dates. If this extended direct aim is deemed unnecessary for particular children, modify the lesson by including only numerals less than or equal to 100, and modify the follow-up by cutting off the bottom portion before giving it to children.

Indirect Aim: to create awareness and inspire curiosity about alternate systems of numeration. to deepen appreciation for ease of operations with a hierarchically based numeration systems

Point of Interest: the riddle-like or secret-code like aspect of conversions

Prerequisites: Exposure to Roman Numerals in everyday life (clocks, etc.) is helpful, but not required.
Alternate Numeration Systems: Roman Numerals

Materials: tickets showing a string of 3, 4, 5, and 6 identical characters in a row (follows)
Chart showing Arabic numerals and their Roman equivalent (follows)
OPTIONAL: A die, a box of dominoes, ideally an extended set of double-9 or higher

Direct Aim: to understand and use Roman numerals that use *subtractive notation* sufficiently well to be able to read numbers up to C (100).

Extended Direct Aim: to understand the Roman system of numeration sufficiently well to be able to read larger numbers such as dates

Indirect Aim: to create awareness and inspire curiosity about alternate systems of numeration. to deepen appreciation for ease of operations with a hierarchically based numeration systems

Point of Interest: the riddle-like or secret-code like aspect of conversions
To experience subitizing

Prerequisites: Facility with Roman numerals that do not use *subtractive notation.*
Exploring Base Systems
Exploring Base 10

Materials: Golden Mat (Decimal Mat) and box of bead bars
1 cm. or 2 cm. Volume Cubes (or other color-neutral tokens)
Tickets for $10^6$ through $10^6$ that dimensionally match the labels 0.0000001
through 1,000,000 on the Golden Mat.

Direct Aims: to explore the decimal system using powers of 10 in expanded notation
to revisit attributes of the decimal system critical to understanding alternate bases
OPTIONAL: to introduce scientific notation. While this is not a standard in the upper
elementary years, by 8th grade, children are expected to know how to perform
operations work with numbers expressed using scientific notation

Indirect Aim: to deepen understanding of the decimal system
to prepare for studies of numeration in bases other than 10

Prerequisites: strong foundation in hierarchies of the decimal system
Success with expanded notation / decomposition
Success with shorthand decimal multiplication (moving the decimal point)
Experience with exponents

Experience with Roman numerals is not a prerequisite. This lesson strengthens
understanding of positional notation – something not used in Roman numerals.
Exploring Numeration Systems
Alternate Base-Systems

Materials:  Golden Mat (Decimal Mat)
            Long black ribbon (length = distance from the units- to the millions-hierarchy)
            Tickets for 10^0 through 10^6 from the previous lesson
            Similarly-sized tickets for 2^0 through 2^6 and for 1 through 64 (base 2)
            Other tickets as desired for other bases (base 16 is fun)
            1 cm. or 2 cm. Volume Cubes (or another color-neutral tokens)
            OPTIONAL: commercial products:
                Multi-base material and chart (all the usual suppliers)
                Multi-base Math Frame (available through ETC Montessori)
                Power of 2 cube (all the usual suppliers)
                Power of 3 cube (all the usual suppliers)

Direct Aims: to explore positional notation in mathematics using numeration systems other than
the decimal (base 10) system

Indirect Aim: to deepen understanding of the decimal system
              To enhance children’s facility with expanded notation and scientific notation

Point of Interest: the riddle-like or secret-code like aspect of conversions

Prerequisites:  Solid foundation in hierarchies of our decimal system.
                Demonstrated ability to think flexibly
                Success with exponential notation in base 10

PRESENTATION ONE: Context and Hierarchies (Counting) in Binary

PRESENTATION TWO: Converting Between Binary and Decimal