Making Sense of Percents



Third Edition by Betsy A Lockhart

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Thoughts Before Beginning Percents

Percentage is an interesting topic. Percents permeate our lives, especially in business. But familiarity has not resulted in comfortability. As a culture, we get the gist of what percentages are trying to tell us without really understanding them. Look around the next time you are in a restaurant to see how many people are reliant on a calculator or a printout on the ticket to know how much to leave as a tip. How common it is for people shop for items and assume that the highest *percent* discount is the best price, simply because they cannot mentally estimate the sale price? And if you want to see people's eyes glaze over, start talking about a 35% increase over a previously measured quantity!

More is the pity. Percentages are not overly complex. Unfortunately, percent operations are classically taught almost as an afterthought, predominantly (or exclusively) through converting percentages to and from decimals. While this is expeditious when teaching about percents abstractly, for Montessorians who foster understanding the concepts that drive the operations (valuing process over product), it is an undesirable shortcut. This shortcut is, I believe, a significant contributor to percents being so poorly understood by the general public.

Montessori said that to teach a thing in isolation is to create confusion. My recollection of learning to calculate with percents is a perfect illustration of this. I well remember being told to *move the decimal point 2 positions to convert from a percent to a decimal*, and then to *multiply by the resulting decimal* to get the right answer. For years I struggled to remember which direction the decimal point was to move. Once had I reasoned out that if the percentage was less than 100%, I would end up with less than I started with, I had a fool-proof way to tell if I had done the problem right – if I remembered to check. (Math was not my long suit in elementary or middle school!)

We can do better than that for our children! We can provide mathematical and practical context that is relevant to the children's prior learning and to their life experiences. The sequence of lessons in this section leads children understand percentages through language, then fractions, and then decimals, applying them through story problems and graphing.

This multi-faceted approach provides *repetition in disguise* through the *spiral curriculum* – brilliant aspects of The Montessori Method. Some of the benefits include:

- strengthening mathematical concepts previously taught in isolation and in application: fractions, ratio/proportions, and/or decimals
- modeling and encouraging flexibility when approaching mathematical problems. *There is rarely only one way to solve a problem!*
- providing relevance a richer "tapestry" of numbers and their operations.

This approach offers the guide a sequencing choice of two different "spiral staircases".

One option is to teach both the fraction approach and the decimal approach (the full sequence in this album) *after children are successful with both fractions and decimals*. This approach has the benefit of causing children to reconnect with fractions in a meaningful way after studying decimals, when they have likely been away from fractions for some time.

- Another choice is to study percentages through fraction equivalencies *after children are successful with abstract fraction operations* and save the decimal approach until *after they are successful with abstract decimal operations*. This approach has the benefit of refreshing the concept of percentages after many months (perhaps even a year or more).

In either case, refreshing concepts in the context of a new procedure is part of what transmutes episodic memory to semantic memory – a more durable form of memory.

The choice of sequence is left to the guide. Factors that might impact this decision include:

- The sequence already in place in other classrooms in the school
- The sequence suggested by the standards followed by the school
- Children's individual pacing relative to the standards (For example, if children are to be held accountable for an understanding of basic percentages before they have encountered decimals, they may be highly successful with percents taught through fractions.)

In some cases, particularly if percentages are introduced after children are fully fluent with both fractions and decimals, students may not need the granularity that has been written into these lessons. In deciding whether a particular lesson could be skipped or combined with another, please exercise caution: do not use your adult brain to assess whether a particular lesson is *abstractly* tricky or not. Rather, let your assessment of the children's previously acquired skills and knowledge and the isolated difficulties spelled out for each lesson to be your guide. (Refer to the Chart of Difficulties that follows.) Remember that some of the lessons are designed to reveal a relationship or a way of thinking rather than a shortcut to calculating the answer. If children have demonstrated mastery of the isolated difficulty of the lesson, a good option might be to quickly review the concept and then press ahead to the next concept in the same lesson. If you get part way through a lesson and the children look bored rather than intrigued, ask them if you are presenting content that they feel they already know. If so, segue into the next lesson!

Some Housekeeping

NOTATION FOR "WRITE THIS"

Many topics are presented concretely *with parallel recording*. This does require the children to do some guided logical processing of equations. Throughout the album, there will be indicators of what to write on the board, building a logical flow to a particular conclusion. The spatial distribution of information – the way we write the equations on the board - can sometimes have a dramatic effect on how well children will follow the logic being developed. In this section, when the visual alignment is important, the text will appear like this:

Ninety percent of the children are going on the trip. There are 30 children.

Ninety percent of the children are going on the trip. There are 30 children. 90% x 30 = going Ninety percent of the children are going on the trip. There are 30 children. 90% x 30 = going 90% x 30 = going 90% x 30 = going 100 This illustrates 3 cycles of writing. Initially, write the sentences "Ninety percent of the children are going on the trip. There are 30 children." In the second step of writing, the portion that is already on the board has been grayed out; the new writing converts the word sentences to a number sentence, writing "90% x 30 = going" so that x is below of, = is below *are*, etc. In the third step, again, the previous writing has been grayed out. The new line converts 90% to 90/100, with all of the elements vertically aligned to the previous sentences. I hope that this provides clarity both for the children and for the guides giving the lessons.

A NOTE ABOUT SOLVING FRACTION EQUIVALENCIES WITH CROSS PRODUCTS

The ability to solve fraction equivalencies using cross products may or may not be a standard to which your children are held at the elementary level. It is a useful tool, one that can give children greater confidence when working with fractions, really coming into play in algebra. (Should you order only this monograph and need instruction, please write for a free copy of that discussion.) Cross-product solutions are included in this section on percents as an option, but it is not a concept that children *need* to understand to be completely successful with the content. It is simply one more way to solve for a missing numerator or denominator. All of the problems within this monograph can be solved with the *multiply by one method*. The choice of whether to include instruction on the *cross-product method* is left entirely to the guide.

FOLLOW-UP OPTIONS

Montessori knew it. Neuroscience supports it. Real learning doesn't happen *in* the lesson – it happens *after* the lesson, when children are working with the concepts on their own. This "second period" of learning, when children follow the process laid out in the lesson, is when they intuit patterns and truths. In this section, each lesson has a provided follow-up that has been designed to lead children through problems or activities <u>of increasing complexity</u>, to develop a deeper understanding of the concept or to use the foundation laid to explore nuances. (More sophisticated problems fall towards the bottom of a section or page.) The significance of this is that if there is a child or children who need less repetition to internalize a process, it is better to have them complete every other problem rather than the top half or bottom half of a page. If, on the other hand, there are children who need to complete problems that are just like the samples shown in the lesson, in some cases, the guide might choose to eliminate the last 2-4 problems in a set, or to make those BONUS problems. The choice is left to the guide, as you know your children best.

The follow-up activities were written presuming that children can complete whole number operations with relative ease. In so doing, follow-ups refresh whole number operations while learning about percents. If this is not the case, if the mechanics of completing the whole number operations is impeding the child's flow, the guide might choose to Isolate the Difficulty, allowing the use of a calculator on some problems, until the concept and the process have been solidified.

In many cases, the provided follow-up will be sufficient practice for children to be prepared for the isolated difficulty in the next lesson in the sequence. If this does not prove to be the case, there are ample resources for additional problems in textbooks or on the internet. Please be sure to preview the problems in light of the isolated difficulty of the lesson prior to giving them to the children, to be sure that these supplementary problems do not presume knowledge that the children do not yet have. Ease of computation is not always the best indicator. This truly is the adult challenge: to view follow-up options through the eyes of your children.

Table of Direct Aims (Isolated Difficulties)

For Use in Decisions About Skipping / Combining Lessons

	Lesson	Direct Aims
Concept	Percentages in Everyday Life †	Define <i>percent</i> Understand percent as a ratio Develop awareness of the use of percentages in real life
	Representing Percents as Fractions	Percent is a fraction out of 100 Experience percentages concretely
	Equivalent Fractions Review	Preparation to solve percent problems with equivalent fractions OPTIONAL: Solve equivalent fractions with the cross- product method
	Expressing Ratios as Percents	Convert to a ratio out of 100, then to percent
Fractions	Calculating Percent of a Known Quantity	Solve for a given percent of a known quantity using fraction equivalencies
	Calculating Percent from a Known Ratio	Calculating percent equivalency of known ratios (fractions) by taking a fraction of 100%
	Applied Percentages	Setting up and solving percent problems from stories Introducing terms <i>percent</i> , <i>base</i> , and <i>part</i>
	Mental Math - Common Percentages † (solving for <i>part</i> or <i>percent</i>)	Use fractions to mentally solve for <i>part</i> when the percent required can be converted to a simple fraction Increase children's confidence through mental math
	Decimal Multiplication and Division Review	Reviewing decimal hierarchy and multiplication and division by 10 and 100 to insure automaticity for upcoming calculations
	Decimal Equivalents to Percents	Convert percentages to decimals Convert decimals to percentages Equate these to common fractions
Graphs Decimals	Calculating Percent of a Known Quantity	Employ decimal equivalents to percents when calculate <i>part or percent</i> . Evaluate a problem to see whether the fraction or decimal method is computationally simpler.
	Applied Percentages	Apply more sophisticated computation techniques to percent situations/stories
	Mental Math – Common Percentages †	Use decimals to mentally solve for <i>part</i> when the percent required is a multiple of 10% or 1% Increase children's confidence through mental math
	Estimating with Percents †	Estimating by rounding percent and/or base
	Solving for <i>base</i>	Using algebraic skills to solve for base
	Interpreting Pie Charts †	Understand the merit / appropriate use of pie charts Interpret data from pie charts as relative quantities
	Creating Pie Charts	Using percent calculations to convert absolute quantities to relative quantities and graph on a pie chart

† strong Practical Life crossover opportunities

Percents as Fractions Representing Percents as Fractions

Materials:

100 board with all tiles in place, numerals face-down so that numeric sequence not needed (prepare ahead)

Hundred Square Paper Fraction Insets Montessori Centesimal Frame and Montessori Protractor White board, markers, eraser

Direct Aim:

Introduce representing percent, "for each 100", as a fraction (hundredths) Experience percentage concretely

Indirect Aim: to prepare children to perform percent calculations

Prerequisites: Understanding percentage as a ratio – a quantity relative to 100.

Presentation:

- Bring out the Hundred Board. Ask the children to reiterate what was shown in the previous lesson with this material. <We showed the meaning of percentage as a quantity relative to 100.>
- Write 13% on the white board and ask for a volunteer to use the Hundred Board to illustrate 13%. <Child removes 13 tiles from the frame. This is 13 tiles out of 100 OR 13 per hundred OR 13% of the tiles.>
- 3. Explain that another way to represent the quantity 13 out of 100 is as a fraction: <u>13</u>

Write $13\% = \underline{13}$ and explore these as two representations of the same quantity. 100

4. Hand the Hundred Board to one child and the marker to another. Ask them to illustrate and record 21% as a percent and as a fraction. <The child writes $21\% = \frac{21}{100}$, showing two ways 100

100

of expressing 21 out of 100.>

- 5. With 2 new volunteers, repeat the process for 50%. <The child writes $50\% = \frac{50}{100}$ showing two ways of expressing 50 out of 100.>
- 6. Look at the fraction 50/100 and its representation on the Hundred Board thoughtfully. Ask if anyone can think of another way to represent that quantity. <It is the same as 1/2.> Look at the way that the child with the Hundred Board chose to represent 50%. Are all of the 50 tiles that remain in the frame on the same half of the frame (all in the top half, the bottom half, the right half, or the left half). Through discussion, build consensus that the 50 tiles arranged cover 1/2 of the area of the Hundred Board no matter which side of the board is filled AND that it is a graphical representation of the quantity 1/2.

- 7. Re-arrange the 50 tiles so that they are spread throughout the frame. Ask these questions: Is it still true that <u>50% of the tiles</u> are on the board? <Yes; no tiles were added/removed.> Is it still true that <u>1/2 of the tiles</u> are on the board? <Yes; no tiles were added/removed.> Is it still true that this is a good graphical representation of <u>1/2 of the frame</u>? <It is not.>
- 8. Bring out a Hundred Square paper. Demonstrate different ways of shading squares to graphically represent the quantity 1/2:
 - Shade the top half
 - Rotate the page so that the bottom half is shaded
 - Rotate the page so that the right half is shaded
 - Rotate the page so that the right half is shaded
 - On a new hundred square, draw a diagonal line from one corner to the opposing corner. Shade 1/2 of the page. Ask if this is a graphical representation of the quantity 1/2. <It is, but it cannot be replicated with the tiles unless we cut some in half diagonally.>
- 9. Reconfigure the tiles on the Hundred Board so that they fill 1/2 of the frame. Carefully set the board on the side of the rug exactly as configured for use later in the lesson.
- 10. Bring out the fraction insets. Choose the 1/2 piece. Confirm that the piece represents 1/2 of the whole circle.
- 11. Lay the various representations of 1/2 side-by-side on the rug: half-filled Hundred Board, two hundred squares previously shaded to show 1/2 of the full hundred square, and the 1/2 circle inset. All of these represent 1/2. Are they equal quantities? <They are not. The semicircle is half of the circle, whereas the tiles are covering half of the Hundred Board. They are each half of a whole, but because they are half of different wholes, they are not equal quantities, just as ½ of a dozen is not equal to ½ of a gross.>
- 12. Again, choose the 1/2 inset. Ask what the piece represents. If children reply with 1/2, prompt them to say "1/2 of the circle" or "1/2 of the unit circle" or "1/2 of the fraction circle."

Ask if anyone can say what percent of the unit circle is in the piece you are holding. <50%>

- 13. Bring out and introduce the Centesimal Frame. The word *centesimal* means *relating to division into hundredths*. Show that the full unit circle has been divided into 100 equal divisions. Children may remember the Montessori Protractor, which divides the unit circle into 360 equal divisions. We call every division on that circle *1degree*. Ask if anyone can say what every division on the Centesimal Circle might be called?
 < Every division is 1% of the circle!>
- 14. Erase the white board. Place the 1/2 inset into the Centesimal Frame and demonstrate reading the frame.

15. Tape the shaded Hundred Square paper to the white board and write/draw the following:



16. Discuss the results with the children. "What do these drawings show?" <They show that 1/2 of something is the same as 50% of it.>

Add to the illustration by writing: $\frac{1}{2} = 50\%$

17. Choose the 1/3 fraction inset and place it in the Centesimal Frame. Demonstrate that it doesn't measure very evenly. It is between 33% and 34%. Return the 1/3 inset to its frame.

Choose the 1/4 fraction inset. Demonstrate that it matches up with 25%

Follow-up:

Children should practice finding the percent equivalent to commonly used, easily conceptualized percentages (follows).

Control of Error:

Teacher-check. (Shadings will vary enough that no CoE card is provided for the shading portion.)

Representing Percents as Fractions

Shade the hundred-squares and unit circles as indicated. Count the squares on the hundredsquare to find the percent equivalent for each fraction. Use the Centesimal Frame to measure the percent equivalent for each fraction. Do they agree? Complete the number sentence in the box telling what you found. The first one is done for you:









Control of Error for Representing Percents as Fractions

Shade the hundred-squares and unit circles as indicated. Count the squares on the hundredsquare to find the percent equivalent for each fraction. Use the Centesimal Frame to measure the percent equivalent for each fraction. Do they agree? Complete the number sentence in the box telling what you found. The first one is done for you:



Shading will vary





 $\frac{1}{5}$ of Hundred Board =











Shading will vary



 $\underline{1}$ of Hundred Board = 10

 $\frac{10}{100}$ tiles = $\frac{10}{\%}$ %



 $\underline{3}$ of Hundred Board = 4





 $\frac{2}{2}$ of Hundred Board = $\frac{2}{2}$

 $\frac{100}{100} \text{ tiles} = \frac{100}{100}\%$



 $\frac{3}{4}$ of the Unit Circle = $\frac{75}{4}$ %



Graphical Representation of Percents Interpreting Pie Charts

Materials:

Examples of Data Tables, Bar Graphs, and Pie Charts (samples follow – prepare ahead by printing, cutting apart, and laminating)

Direct Aim:

To interpret data from pie charts as expressing relative quantities

Indirect Aim:

Preparation to create pie charts

Prerequisites:

Fundamental understanding that percent expresses a ratio, a relative quantity Understanding that 100% means *all*

Experience with several other types of graphs is extremely helpful, but not required (Children should have at least experienced bar charts)

Presentation:

- 1. Initiate a discussion about children's prior experience with graphs. Key points to include:
 - graphs visually depict data to tell a story
 - numbers can tell the same story, but it takes longer to interpret numbers than a picture
 - Discuss the quote "A picture is worth a thousand words" applies to graphs.
- 2. "Let's look at an example. Let's pretend that we took a poll in our class to see how many pets of each type the children in our class have." Show the tabular results that follow this lesson.
- 3. Most graphs show how numbers relate to each other. A bar graph or histogram would show us these results nicely. Show the bar graph. Discuss the relative merits of the bar graph to the raw data.
- 4. Bar graphs show the absolute magnitude of the numbers. We can see exactly how many dogs, cats, fish, ferrets, and horses our class has altogether. But sometimes we want to see the quantities relative to one another. For example, what if we wanted to know if the number of dogs and cats made up more or less than half of all of the pets owned by our children. How would we do that with a bar graph? <We would have to add up the number of dogs and cats and also the total number of pets in the class. Then we would have to compare the two numbers.>

- 5. An alternative is the pie chart, which shows the numbers as percents. Show the pie chart. Explain that the blue segment represents the percent of class pets that are dogs. The red shows the percent of class pets that are cats. The green... (etc.). With this graph we can easily see how the number of pets that are dogs and cats compare to the total.
 - Ask what it would look like if the number of dogs and cats was exactly half of the total number of pets. <The blue and red sectors together would make a semi-circle.>
 - Ask if the number of dogs and cats as shown on this pie chart is more or less than half. <It is more.>
- 6. Compare the pie chart and the bar graph. Both show that dogs are the most popular, followed by fish, cats, horses, and ferrets.
 - Ask which would be more useful if we wanted to know the exact number of horses.

bar graph>
 - Ask which would be more useful if we wanted to know how fish, ferrets, and horses combined compared to dogs? <The pie chart it shows that the number of fishes, ferrets, and horses combined are roughly equal to the number of dogs. In point of fact, the data says that the combined total of fish, ferrets, and horses is 31 1 less than the number of dogs. We can't see that quite so precisely on the pie chart, but we can say that they are about equal.>
- 7. Sometimes pie charts have a "data call-out". Show the pie chart with data call-out. Discuss how this makes the comparison of segments a bit more precise we total the percentages for fish, ferrets, and horses and see that together they represent 40% of the pet population. When we compare that to the percent of the pets that are dogs, we can see that there are more dogs by 1%.
- 8. Show the pie chart <u>Sources of Electricity Generation in Iowa in 2016.</u> (https://commons.wikimedia.org/wiki/File:Iowa Electricity Generation Sources Pie Chart.svg.) Ask pertinent questions such as which is the predominant source for electricity, how the three smallest sources combined compare to coal or wind, etc. Round each percent off to the nearest 5%. Coal provided about 45% of the electricity for Iowa; wind provided about 35% and nuclear provided about 10%. With those approximations, we can see that coal produced about 4 ½ times as much energy as nuclear; wind provided about 3 ½ times as much electricity as nuclear.

Follow-up: Children should interpret data presented on pie charts. A sheet with a few graphs is provided for an initial experience, but pie charts will take on more significance if the children see real data on topics of current interest being conveyed graphically. USA Today is a great source for pie charts – they typically have at least one displayed prominently each day. If interpreting pie graphs can become a day-starter activity at least once per week, current events and mathematics will be integrated, providing relevance and cross-curricular applications (curricular webbing).

Control of Error: One is provided for the activity sheet that follows.

Pets in Our Class

Dogs and puppies	32
Cats and kittens	15
Fish	22
Ferrets	2
Horses	7









Interpreting Pie Charts

The pie chart to the right shows **Transport Usage in Singapore** how people get around in Singapore. Please answer the following questions: 2096 1. What form of transportation is most popular? Buses & Trains E Cars Taxis 2. What relationship can you see between cars and taxis combined and busses and trains combined? http://singaporetransportpolicies.blogspot.com/ 3. Can you compare people's use of busses and their use of trains? Why or why not?



The pie chart to the left shows relative amounts of CO₂ emissions generated per country. Please answer the following questions:

- 4. Where does the US rank on this chart (first, second...)?
- 5. What country produces about half as much of the CO₂ emissions generated in the US?
- 6. China produces about how many times more

emissions than the US?_____

The pie chart to the right shows children's favorite healthy snacks. Please answer the following questions:

 If the total number of children polled was 240, how many children replied that they

preferred cheese? _____

8. How many children preferred hot dogs?

Favorite Healthy Snack



Control of Error for Interpreting Pie Charts

The pie chart to the right shows how people get around in Singapore. Please answer the following questions:

- 1. What form of transportation is most popular? Busses and Trains
- What relationship can you see between cars and taxis combined and busses and trains combined?
 <u>Cars + Taxis is about equal to</u> Busses and Trains



http://singaporetransportpolicies.blogspot.com/

3. Can you compare people's use of busses and their use of trains? Why or why not? No comparison can be made because they were combined into a single category or response.



The pie chart to the left shows relative amounts of CO₂ emissions generated per country. Please answer the following questions:

- 4. Where does the US rank on this chart (first, second...)? <u>The US is third, behind China and all other countries combined.</u>
- What country produces about half as much of the CO₂ emissions generated in the US? <u>At about 7%, India produces about half</u> <u>American production</u>
- 6. China produces about how many times more emissions than the US? <u>China at ~30%.</u>, <u>produces about twice the emissions of the US.</u>

The pie chart to the right shows children's favorite healthy snacks. Please answer the following questions:

- 7. If the total number of children polled was 240, how many children replied that they preferred cheese? 20% of 240 = 48
- 8. How many children preferred hot dogs? <u>1% of 240 = 2.4 Since you can't have 2.4</u> <u>children, the answer is 2 or 3 children.</u>

Favorite Healthy Snack

