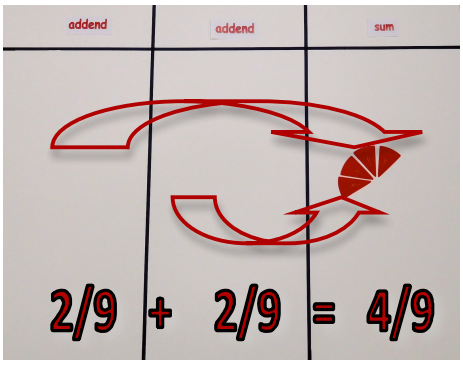
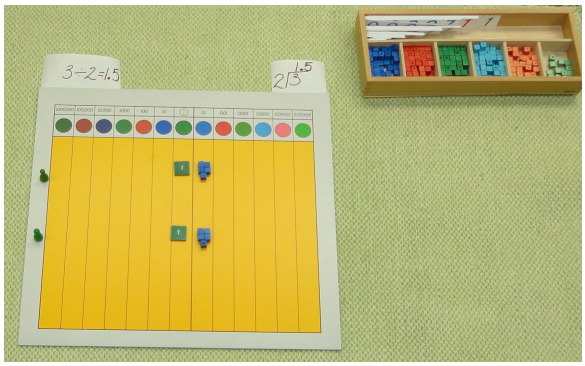
Zero to One

Thought-provoking Activities and Problem Sets for Truly Knowing Fractions and Decimals



by
Betsy A. Lockhart



For information address:
30472 Middleton Road, Evergreen, Colorado 80439

Your Comprehensive Fractions and Decimals Material Making Masters

Greetings, Mathmagicians!

The following pages include everything the informed Montessorian needs to unlock the mysteries of Fractions and Decimals. No special incantations are needed to needed to unlock these mysteries – only the faithful following of materials-based instruction with lessons and activities that isolate the difficulty for the child.

It is the emphasis on isolated difficulties that has been missing. Most Montessori training does an admirable job of isolating the difficulty in presentations, but the available follow-up problem sets and activities have been taken from or modeled on those designed for traditional education. Most of these resources place emphasis on computational simplicity rather than on illuminating aspects of the process. There are two issues that arise from using these problems:

1) Many problems exceed the capacity of Montessori fractions materials, pushing teachers to push children to abstraction prematurely.
2) All problems from a given operation (like non-common denominator fraction addition) are treated as being alike. In reality, especially when teaching concretely, there are often different types or cases of problems within a single operation. For example, there are 3 *types* of non-common denominator problems.
    a. The denominator of one fraction is a factor of another, such that the larger denominator **is** the LDC (i.e. 1/2 + 1/4 → the LCD is 4).
    b. The denominators have no common factors, such that the LCD **is the product** of the two denominators (i.e. 1/3 + 1/4 → the LCD is 12).
    c. The denominators have a common factor, such that the LCD **must be researched** (i.e. 1/4 +1/6 → the denominators have 2 as a common factor; the LCD is 12).

When children are led through the 3 types of denominators as 3 different cases, it gives them a strategy for solving problems abstractly. Children can be taught to look for which *type* of denominator pair is present, to streamline the conversion process and make it more accurate and methodical.

These material-making masters serve both as a supplement to whatever Fractions and Decimals Album you received in training, and as a review of scope and sequence. The **Reproduction Guide** is a "quick start" table to get your cards printed correctly on the first try.

Following the **Reproduction Guide** is a section that describes the different types of materials that are included in this packet, with some considerations for reproducing, using, and storing these materials.

In the **Scope and Sequence** section, the isolated difficulty is called out for each Task/Command Card, Problem Set, or Activity. If at any time, the child shows a need for additional practice with a particular concept, the teacher can look to the Table of Contents to see what difficulty was being isolated, and can create more problems for the child of the same type.

The last section includes some **Final Thoughts** on organizing materials and on the limitations imposed by the materials.

We hope that these materials will help you make mathemagic in your classrooms, delighting you and your children for years to come!

-Betsy Lockhart

Contents of your Material Making Masters

REPRODUCTION GUIDE

These pages portray the different sections of this product and how they are to be reproduced and/ or assembled.

REVIEW OF FRACTION OPERATIONS

These 4 pages are a reminder of the basic choreography of each of the four fraction operations. No matter what isolated difficulty is being examined, the basic choreography of the materials remains the same. This is included as a teacher reference; however, some may find that printing these 4 pages on card stock and posting them in the room provides a useful resource for the children as well.

COMMAND CARDS

These cards contain activities for children that are impressionistic or experiential. These activities generally precede operations work and significantly contribute to children's understanding of the concepts being revealed in the operations work. Unlike the Problem Cards, many of these activities have no provided control of error – an adult or an older child checks them visually for completion and accuracy.

Many of the command cards have 2 or 3 different activities listed. Since the activities generally support the same isolated difficulty, it is left to the teacher / guide to decide whether to offer the children the option to choose just one of the activities, only doing additional activities only if they are inspired to do so.

Occasionally there are several cards that support a single isolated difficulty. This can be seen 2 ways: listed in the **Scope and Sequence** section and indicated on the cards themselves. See, for example, Fractions Command Cards G through K. All of these cards support explorations related to understanding how numerators and denominators work together. The second line on each card indicates this, but also please note that the number of the activity increments from 1 on card G to 7 on card K. *It is not to be inferred that every child needs to do all 7 activities on the 5 cards*! It is left to the teacher to decide if specific activities will be suggested or required of the children, or if the children can look at the cards and choose activities from among those 7.

CHARTS and ACTIVITY SHEETS

These pages are charts to be used in lessons, for display purposes, or as paper handouts for lessons and/or follow-ups as specified in the Reproduction Guide above and the Scope and Sequence chart below.

CONTROLS OF ERROR

These cards contain solutions for all of the equations in both the Command/Task Card Sets and Problem Card Sets. Note that many of the early Command/Task Cards do not have a Control of Error – an adult or older child checks them visually for completion and accuracy.

FRACTIONS AND DECIMALS TAG LABELS

These labels (being mailed to you) include most or all of the labels that are useful for Fractions and Decimals lessons and follow-ups. They have been designed to adhere to card stock before laminating. Refer to the Reproduction Guide for color, size, and storage suggestions.

Notes for Fraction and Decimal Tags

<u>Operations Nomenclature</u>: Names for parts of a multiplication problem are provided using both the traditional vocabulary (multiplicand, multiplier, and product) and the more contemporary

vocabulary (factor, factor, product). Each set of terms has advantages and shortcomings. It is recommended that the teacher/guide use the terms that are employed in whatever standardized testing is used at the school. A best practice is to introduce both sets of terms and to use them interchangeably.

Decimal Hierarchy (Names): Two tickets reading **Ones / Units** *are provided. When labeling the decimal mat, only one ticket is needed. When labeling the Decimal Checkerboard, 2 are needed.*

Decimal Numerals: These tickets will be used for Task Card E: Reading Whole and Decimal Numbers. They may be used any other time as a random number generator.

Decimal Checkerboard Labels for individual squares: These labels fit 4"x4" foam or felt squares if little or no cardstock frame is left around the label. Mount on black (or another neutral color) with label from left column on one side, right column on reverse. DO NOT mount on hierarchical colors.

PROBLEM CARD LABELS

These labels (being mailed to you) contain equations to be solved. They have been designed to adhere to card stock before laminating. Refer to the Reproduction Guide for color, size, and storage suggestions.

Problems have been meticulously designed for use with Montessori materials. Cards isolate the difficulty of implementing the process with materials, as noted in the Scope and Sequence table that follows. For example, when adding non-common denominator fractions, as mentioned in the introduction, there are 3 different cases of denominators:

- denominator of one fraction is a factor of the other (LCD is the larger denominator)
- denominators have no factor in common (LCD is the product of the 2 denominators)
- denominators have a common factor (LCD must be researched)

The teacher/guide can give a single lesson on the process of renaming fractions so that they have common denominators and then ask children to complete the first set of problems. If they are successful with those, they can go on to the next set, often without another lesson or with minimal additional instruction (perhaps just drawing attention to how the new problem type / case differs from the previous ones).

Sometimes there is just one card illustrating a particular difficulty but sometimes there will be more than one. When more than one card of a particular type is provided, in the estimation of the author, more repetition is typically needed to internalize the concept. This is all spelled out in the Scope and Sequence table that follows, but can also be noted by the naming of the problems. Each time a new difficulty is integrated, the problem "numbering" resets to A. Thus, if a card has problems E through H, it is a continuation from the previous card. Generally speaking, this indicates that the author suggests that children complete both cards (problems A through H) in a single sitting; however, it is left to the discretion of the teacher/guide how much is appropriate for each child.

Note: It is presumed that children are getting math practice (although not necessarily a math lesson) on a nearly daily basis. If children engage with math only once per week, they will likely need more repetition than is provided in this material. The teacher can provide more problems of the type being practiced by referring to the isolated difficulty for that problem set in the Scope and Sequence Chart.

Notice that at the beginning of each section there is a label card that announces the operation type of the following problem cards. These could be used to label drawers or to label a divider within a box like a recipe box.

Sample Scope and Sequence (first page shown)

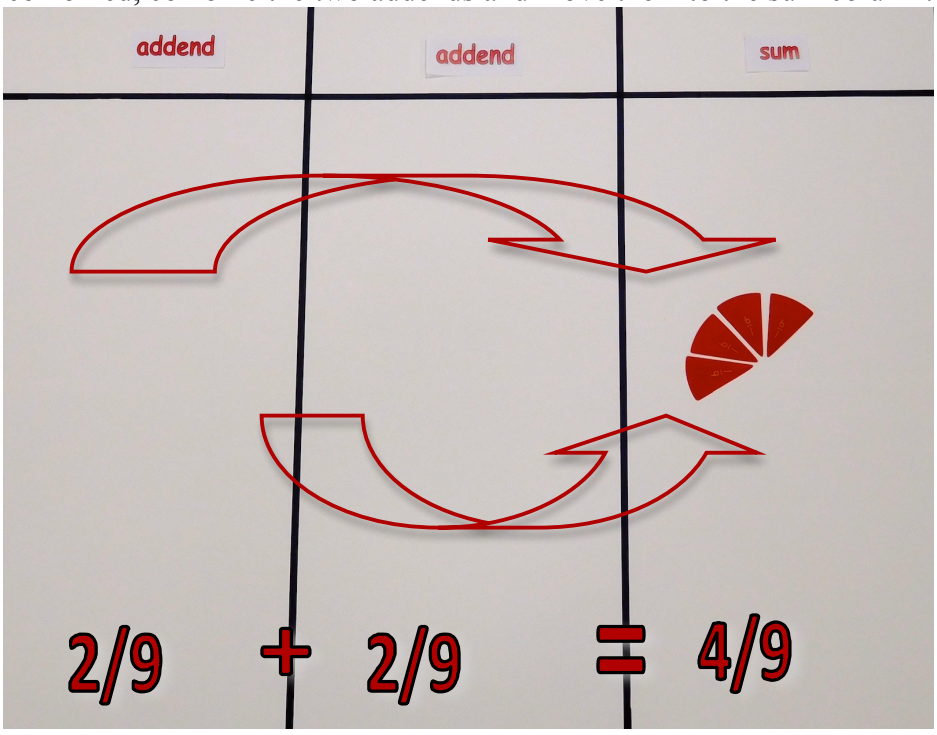
These pages define the concepts being taught in sequence and the isolated difficulty in each command/task card and each problem card set. The list also shows where specific handouts, charts, and labels are to be used.

FRACTIONS **Label, Chart, or Activity Sheet**	**Isolated Difficulty**
Introducing Fractions, Notation, and Equivalent Fractions	
Task Cards A–D: Introducing Fractions Metal Inset Labels	Fractional Parts of Everyday Objects (A) Fractional Parts of Metal Inset Paper (B) Introduction to Fraction Circles (C-D)
Task Cards E–K: Fraction Notation Number Line Activities 1&2	Family Names: Denominators (E) Pieces Used: Numerators (F) Numerators and Denominators -magnitude of denominator vs. the size of the piece (G) -expressing fractions of a collection (H) -representing fractions through drawings (I) -abstract recording & concrete relative magnitude (J-K)
Task Cards L–O:Equivalent Fractions Mute Chart of Equivalent Fractions Control Chart of Equivalent Fractions Hundred-Square Paper (9 sq./child)	Finding Equivalents (L) Chart of Equivalent Fractions (M) Changing Improper Fractions to Mixed Numbers (N) Fractions on Hundred-Square Paper (O)
Addition/Subtraction Common Denominator Fractions	
Problem Cards 1-30 Operations Labels	Sums ≤1 – static: no reducing needed (1-3) Sums ≤1 – dynamic: sums need reducing (4-5) Sums >1: fractions sum to Mixed Numbers (6-8) Sums >1: addends are Mixed Numbers –Static (9-10) Sums >1: addends are Mixed Numbers –Dynamic(11-14) Static Fraction Subtraction (15-17) Dynamic Fraction Subtraction (18-20) Static Mixed Number Subtraction (21-23) Dynamic Mixed Number Subtraction (24-25) Dynamic Mixed Number Subtraction with Exchanging (26-30)
Addition/Subtraction Non-Common Denominator Fractions	
Task Cards P–Q: Adding Non-Common Denominator Fractions (Exploration Stage)	Adding Fractions with Different Denominators Case 1: Larger denominator is the LCD (P) Case 2: LCD is the product of the 2 denominators (Q) (no factors in common)
Task Cards R–W: Multiples Multiples Sheets Common Multiples Sheets	Concept of Multiples (R) Research of Common Multiples Concept of Common Multiples and Prime Numbers (S) Least Common Multiple with Paper (T) Least Common Multiple with Pegs (U) Least Common Multiple – 3 Factors (V) Least Common Multiple – 2-Digit Factors (W)

Sample Review of Fraction Process (addition shown)

1. Load the first and second addends into the first two columns:

2. After whatever exchanging is needed to allow the two fractions to be combined, combine the two addends and move them to the sum column.



3. Simplify the sum if needed and record.

Sample Task / Commands (copy to card stock)

Introducing Fractions C
III: Introduction to Fraction Circles

1. Use the Fraction Inset labels to give each metal inset a title (halves through tenths). Ask an adult or older child to check your work and ask you these questions:
- How many <u>equal</u> <u>pieces</u> are there in thirds?
- How many <u>equal</u> <u>pieces</u> are there in fifths?
- Ninths have 9 pieces. What is special about the 9 pieces?

2. Make or add to a poster / booklet titled **Fractions are Equal Parts** using the metal fraction insets for halves, fourths, and eighths. Can you find a way to show sixteenths?

Addition/Subtraction of Non-CD Fractions T
III: Research of Common Multiples

2. Obtain a few copies of the <u>Common Multiples</u> paper. Choose 2 numbers for which you want to find the Least Common Multiple (LCM). You may choose from those listed below or make up your own. Then do this:
   - Write the 2 numbers at the top of the page.
   - Circle all of the multiples of each of the two numbers up to 100.
   - Write a list of the common multiples, the numbers that are circled twice, near the bottom of the page.
   - Put a bold square around the smallest number in that list, the Least Common Multiple (LCM), and write that number at the bottom of the page.

A. <u>2 and 3</u> B. <u>3 and 4</u> C. <u>3 and 5</u> D. <u>3 and 7</u>

Introducing Decimals & Their Notation K
VI: Comparing Composite Numbers

2. Build each pair of numbers on the Decimal Mat. Decide which quantity is larger. Then write the equation with an less than (<) or greater than (>) in the circle.

- 145.89 ◯ 145.98
- 53.12 ◯ 51.32
- 54 ◯ 53.9999
- 0.0056 ◯ 0.00065
- 1,000.5 ◯ 999.7895

In your own words, how would you describe how to decide which of 2 numbers is larger?

Decimal Fraction Multiplication S
V: Labeling the Decimal Checkerboard

1. Alone or with a friend, lay out all of the double-sided Decimal Checkerboard Labels so that the product (units) is face down and the equation (units x units) is face up. Arrange the labels in a 7 x 7 grid so that they are in a random order. Youngest player goes first, turning over two tickets that he/she thinks have an equal product. If the products match, the player keeps the pair of tickets. If not, the tickets are flipped back to their original orientation. In either case, play passes to the other player.

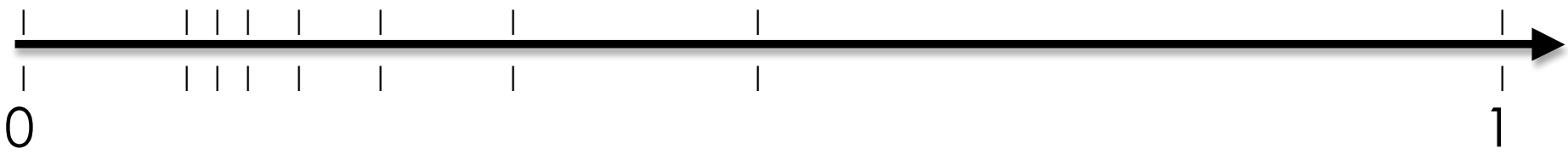
The game is over when 7 tickets remain without a match. Turn all 7 tickets over and put them in order from largest to smallest.

Sample Charts / Activity Sheets (copy to paper or card stock)

Number Line Activity 2

Collect the fraction pieces inside the box, below, and order them from smallest to largest. Write their values on the number line in the correct spaces. (You may need to lengthen some the marks to be able to fit in all of the fractions!) :

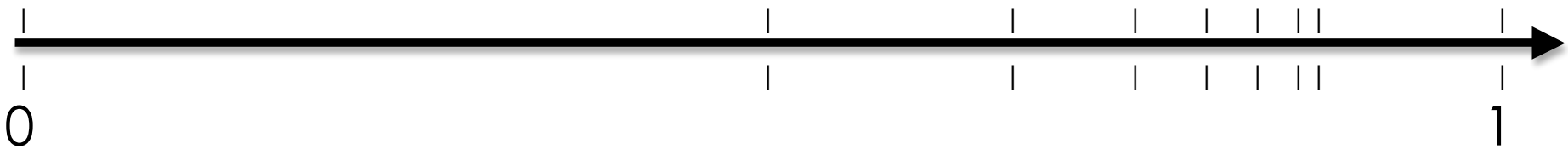
$\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{7}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{1}{5}$



0 1

Collect the fraction pieces inside the box, below, and order them from smallest to largest. Write their values on the number line in the correct spaces. (You may need to lengthen some the marks to be able to fit in all of the fractions!) :

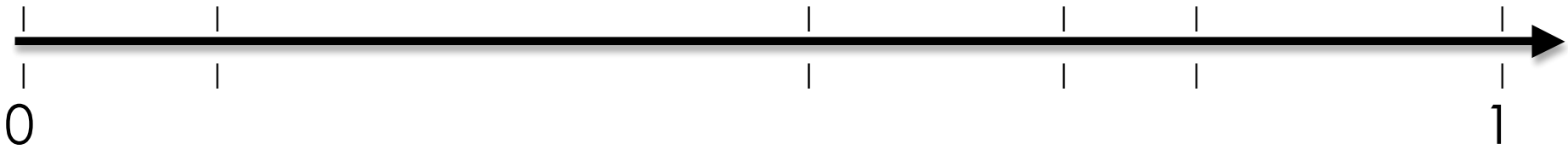
$\frac{2}{3}$ $\frac{1}{2}$ $\frac{6}{7}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{4}{5}$



0 1

Collect the fraction pieces in the box, below. Order from smallest to largest. (Hint: some fractions may be equal to one another!) Write each fraction on the number line.

$\frac{2}{3}$ $\frac{1}{2}$ $\frac{4}{6}$ $\frac{3}{4}$ $\frac{4}{8}$ $\frac{3}{6}$ $\frac{1}{8}$



0 1

Checkerboard with Labels Control

M X U	HTh X U	TTh X U	Th X U	H X U	T X U	U X U
M x t	HTh x t	Tth x t	Th x t	H x t	T x t	U x t
M x h	HTh x h	TTh x h	Th x h	H x h	T x h	U x h
M x th	HTh x th	TTh x th	Th x th	H x th	T x th	U x th
M x tth	HTh x tth	TTh x tth	Th x tth	H x tth	T x tth	U x tth
M x hth	HTh x hth	TTh x hth	Th x hth	H x hth	T x hth	U x hth
M x m	HTh x m	TTh x m	Th x m	H x m	T x m	U x m

Sample Control of Error

Control of Error - Decimals

Introducing Decimal Fractions and Their Notation

I/II. Decimal Mat to Millionths

Task/Command Card A

1. Visual check
2. Friends check each other during the game.

III. Decimal and Whole Number Relationships

Task/Command Card B

Compare to color Candelabrum of Numeration Chart

IV. Creating Decimal Numbers

Task/Command Card C

1. Visual check
2. Friends check each other during the game.

Task/Command Card D

Visual check

V. Reading Whole & Decimal Numbers

Task/Command Card E

1. Visual check
2. 154,367,869.190123
   Visual check of drawing

Task/Command Card F

3. Visual check
4. Friends check each other during the game.

V. Reading Whole & Decimal Numbers

Task/Command Card G

5. 123,456,789.012345
   8 tens are
   5 ten-thousands
   2 ten-millions
   0 tenths
   3 ten-thousandths
   9 units
   There is 1 hundred-million and 1 hundredth.
   There are 3 millions and 3 ten-thousandths.
   There are 5 ten-thousands and 5 millionths.

Task/Command Card H

6. 9,583,107.296487
   Nine million, five hundred eighty three thousand, one hundred seven thousand AND two hundred ninety six thousand four hundred eighty seven millionths!! WOW!

Task/Command Card I

7.

- One thousand one AND one tenth
- Thirteen million, thirteen thousand, thirteen AND thirteen thousandths
- Twenty seven thousand AND four hundred fifty six thousandths.
- One million AND one millionth
- Ninety million, ninety thousand, nine hundred nine AND one hundred twenty three thousand four hundred fifty six millionths!

Sample Fractions and Decimals Tags (on labels to be mailed to you)

Metal Inset Labels	ten tenths
Operations Nomenclature	addend
Decimal Hierarchy (Names)	**Millionths**
Decimal Hierarchy (Wholes & Fraction Notation)	$\frac{1}{1{,}000{,}000}$
Decimal Hierarchy (Decimal Notation)	0.0000001
Powers of 10	10^{-6}
Note to Sneek-a-Peekers: this label and the one opposite it are placed on opposite sides of the same ticket for use with the Decimal Checkerboard. **Units X Units**	**Units**

Sample Problem Cards (on labels to be mailed to you)

Problem Sets

Adding Non-Common Denominator Fractions

Adding fractions & mixed numbers
- with 1 in the numerator
- one denominator is a factor of the other
- denominators have no common factor
- denominators have a common factor

www.lockhart-learning.com

31

+/- Non-Common Denominator Fractions

VII. Adding

Find the least common denominator using the fraction equivalency chart, pegs, multiples paper, or abstractly and solve for these sums.

A. $\frac{1}{3}+\frac{1}{9}=$ B. $\frac{1}{2}+\frac{1}{4}=$

C. $\frac{1}{10}+\frac{1}{5}=$ D. $\frac{1}{2}+\frac{1}{8}=$

Problem Sets

Fraction Division

Division of a Fraction by a Whole Number (f÷w)
- Unit Numerator in Dividend
- Numerator of Dividend Divisible by Divisor
- Numerator of Dividend is a Factor of Divisor

Division of a Whole Number by a Fraction (w÷f)
- Unit Numerator in Divisor
- Non-Unit Divisor Numerator=Factor of Dividend

Division of a Fraction by a Fraction (f÷f)
- Numerators (Dividend & Divisor) are equal
- Dividends' Numerator = Multiple of Divisors'
- Numerators (Dividend & Divisor) are unrelated

www.lockhart-learning.com

97

Division of Common Fractions

IV: Fraction ÷ Fraction

Give the quotient of each equation in lowest terms.

A. $\frac{2}{3}\div\frac{3}{4}=$ B. $\frac{3}{4}\div\frac{2}{3}=$ C. $\frac{2}{5}\div\frac{3}{5}=$

D. $\frac{5}{7}\div\frac{2}{3}=$ E. $\frac{5}{9}\div\frac{2}{3}=$ F. $\frac{2}{7}\div\frac{3}{8}=$

G. $\frac{4}{5}\div\frac{3}{5}=$ H. $\frac{2}{3}\div\frac{3}{10}=$ I. $\frac{5}{6}\div\frac{2}{7}=$

Problem Sets

Multiplying Decimal Fractions

Multiplication as Repeated Addition
Multiplication with Recording Partial Products
Introduction to the Decimal Checkerboard
Building / Labeling the Decimal Checkerboard
Wholes x Composites
Composites x Composites
(Passage to Abstraction)

www.lockhart-learning.com

Decimal Fraction Multiplication 8

I/II. Multiplication as Repeated Addition

Use the Decimal Mat and Skittles with either Pegs/Stamps & Decimal Cubes OR Bead Bars to find the following products. Check your answer with repeated abstract decimal addition.

A) 51.4 x 3 =

B) 647.89 x 4 =

C) 4.627 x 5 =

D) 99.99 x 7 =

E) 0.5974 x 6 =