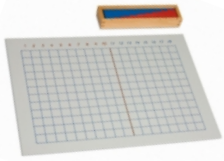
Making Math Facts Memorable

Strategies and sequenced activities
to help children efficiently and
effectively internalize math facts.



by Betsy Lockhart

For information address: PO Box 3443, Evergreen, Colorado 80437-3443

First Edition

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Table of Contents

INTRODUCTION

PART I – PHYSIOLOGICAL INFUENCES: HOW WE LEARN "READY-RECALL" FACTS

PART II – CLASSROOM LEADERSHIP: CREATING A CULTURE OF MATH FACT MASTERY

PART III – LESSONS AND ACTIVITIES

Preface – Making Math Facts Memorable

Math Facts Mastery is elusive!

Everywhere I go, I hear teachers say that it is harder and harder for children to internalize (memorize) math facts. Lower elementary teachers say that children are not as prepared to enter first year as they once were, and they lament that techniques for internalizing facts that have been effective for years now seem to be bearing less fruit. Upper elementary teachers bemoan the fact that every year a smaller percentage of fourth year children have achieved mastery of even addition and multiplication facts by the time they enter upper elementary; these teachers are surprised to find themselves having to teach math facts now. Most people who work with children agree that there are a number of things that have changed for and about children in the past 10 years, many of which could be a contributing factor to slower fact mastery. For example, children have more demands and more opportunities today (both within the classroom and outside school) and are consequently spread thinner than ever before; it is harder for children to *rest* with a material long enough to discover everything that the material has to offer. We also know that children are exposed to more visual stimulation (much of it rapid-fire) at a younger age, when brains are still malleable, shifting children's preferred learning modalities and, in some cases, shortening their concentration span. Recent research shows that practice activities such as those used in both Montessori and traditional classrooms are very important in developing automatic recall; however many children will not spontaneously memorize facts solely from experience with manipulatives nor from problem-solving activities. We must give direct instruction on math facts and number strategies for our children to develop truly automatic recall of facts. If, then, we still value children mastering their math facts, we must take a more active role than we might have in past years.

Why do we care?

Certainly having automatic, accurate recall of math facts better insures correct answers when doing operations work abstractly; it can speed along materials-based operations work; it facilitates the child transitioning from materials-based mathematics to an abstract (paper-pencil) process. *However, these statements belie the far more significant truth that facts mastery is vitally important to a child's comprehension of the gestalt of an operation; "how does this operation work?"*

Researchers tell us that the human mind has a limited capacity to process information, and if too much energy goes into figuring out the math fact value, little energy is left to understand concepts. This means that weak math-fact automaticity can have a devastating affect on the child's ability to understand the process of an operation, let alone to internalize it. It also limits the child's ability to discern broad mathematical patterns. This directly parallels the reading research of the early 1980s, which showed that readers who struggle to decode have poor reading comprehension; they work so hard to decode individual words that by the time they arrive at the end of a sentence, they no longer remember what they worked so hard to decode - - they don't know what the sentence says.

Some argue that a lack of understanding of the process of mathematics can even impact other areas of study, as teachers increasingly attempt to integrate applications and real-world events to increase relevance or in an effort to follow NCTM guidelines. For example, consider a teacher trying to help children relate to history / geography studies by comparing the size of a typical city-state in Ancient Mesopotamia to that of modern day cities, or by comparing historical statistics about rainfall from one geographic area to another. A student who lacks automatic access to math facts and/or has poor number sense may not comprehend the significance of much of the class discussion and so may fail to increase his understanding of the history / geography concepts taught.

What is *automaticity*? How fast is fast enough?
Typically, the goal of fluency / automaticity is generally set at 30-50 correct responses per minute if given orally; 30 facts in 3 minutes if written. That translates to 2 seconds per fact if given orally, and 3.6-6 seconds per fact if written. Try this experiment: have a friend (or a student) go through a packet of flash cards with you where the friend's only goal is to flip one card every 2 seconds, whether you respond immediately or slowly, correctly or incorrectly. Does 2 seconds per response seem like an *automatic* oral response time? Is it too long or too short?
What if the response is written or on a computer keyboard? Certainly that takes longer and can be impacted by things like fine-motor control, as reflected in the longer allotments. Try taking a fact time test and see what your per minute rate is for addition facts. Does 3.6 – 6 seconds per response seem like an *automatic* response time? Is it too long or too short?
What do your instincts tell you about the difference in response time if the respondent is doing 10 facts or 100 in a single sitting?

I prefer to set a goal that is perhaps less measurable, but certainly more meaningful. I tell students that math facts are automatic when they see a math fact and know the answer without having to research it. If I hold up a card that says "6 x 8" they know in their heads and in their hearts that the answer is 48 without having to skip count or recite rhymes or use any other mathematical or mnemonic tricks. If a child (or adult) is able to do this, he will easily respond orally within 2 seconds – and probably much faster!

Some children will not be able to achieve 100% automaticity for 100% of their facts. For these children, mnemonics and other math tricks can help them achieve the time goals listed above. But I prefer that they continue to strive for true automaticity as their growing brains mature.

How can we guide children to automaticity concretely AND efficiently?
We cannot increase the number of hours in a day; neither can we increase the amount of time dedicated to the mastery of math facts without decreasing the time dedicated to some other pursuit. That means that we must use the time we have to greater advantage! We must eliminate unproductive practices (and sometimes long-held beliefs), and adapt practices that are somewhat successful to produce greater benefits. This monograph will help you

- understand how the brain works when learning *ready-recall* facts.
- understand the modalities that humans use to process and store information
- adapt our teaching strategies and practices to simultaneously engage multiple modalities
- adapt our practices to insure optimal duration and frequency of practice
- insure that children understand the goal of automatic fact recall and its importance <u>for them</u>
- engage the children in goal-setting, measuring and celebrating
- individualize activities so that the majority of time is spent on facts still being mastered

Here's the good news: <u>*All of this can be accomplished without significant increase to the teacher's record-keeping or instructional time!*</u> It requires replacing some practices with others that are more efficacious, but these new practices do not need to take more time than whatever you are doing in your classroom right now!

Intrigued? Then turn the page to step into a short summary of brain function and learning modalities, leading to discussions of ways to use familiar Montessori materials and other didactic materials and activities to better enable children to internalize math facts!

PART III – LESSONS AND ACTIVITIES
Introduction: Highlights and Themes

Highlights

The following sections step through each of the four operations with a sequence of lessons and activities for the classroom and for home. The sequence includes traditional Montessori lessons and materials, as well as other activities and resources. In some cases, there are suggestions for ways to slightly alter or augment the traditional presentation or the students' use of materials to optimize facts retention from the activity. *The greatest level of detail in the discussion (implementation and rationale) is in the sections for addition or multiplication facts.* When encountering the same material / concept in later sections, you will see a reference to the correlative addition or multiplication section.

In this monograph, addition is followed by multiplication rather than subtraction; this is a traditional Montessori approach. I am a strong proponent for this sequence. Multiplication is repeated addition, whereas subtraction is the inverse (opposite) operation to addition. The supporting logic is as follows:

- Multiplication and addition both "build up".
- Multiplication facts reinforce addition facts. Initial attempts to memorize multiplication facts include skip counting, which is simply serial addition by a common addend.
- Subtraction, as the inverse operation, is mathematically opposite to addition and multiplication. It requires an entirely different thought process and a different process when manipulating materials.
- Multiplication facts through x5 are so much easier to master than subtraction through -5. Neither is particularly easy for 6s and beyond, but even there, multiplication is easier for most than subtraction. Having children tackle multiplication facts before subtraction facts builds a child's strategic approach to learning facts and gives them greater success, and thus, greater confidence and willingness to take academic risk when pursuing later facts.

Should you (or your school) choose to teach subtraction facts after addition facts rather than multiplication, there will be some activities and homework that will need to be altered or resequenced. It will be completely evident which need to be changed as you work through the process; I call it out here in the introduction simply to raise awareness for those choosing an alternate sequence.

Themes

There are six themes that play out in the following sections. These themes reflect key learning strategies, some of which were discussed in Part I and/or Part II of this monograph. Your understanding of why the lessons and activities are arranged as they are in the following sections will be greatly enhanced if you spend a few minutes reflecting on these themes before tackling the specific lesson / activity sequences. The themes are discussed in greater detail with examples in the following sections.

Theme One: Learning Math Facts is different from practicing or drilling or testing

Math facts practice sheets, traditional flash cards activities, and most math facts software *drill* or *test for math fact knowledge already internalized.* This has its place, but does not *teach* math facts and is fairly ineffectual at internalizing facts not yet mastered. It is vitally important that we not confuse the two and that we use each at the proper point in the sequence.

Theme Two: Authentic learning activities focus exclusively on facts not yet mastered

Many activities (paper-pencil, computer based, and Montessori materials based) exercise all of the combinations within a given operation. This is appropriate for children practicing previously mastered facts for retention, but less desirable for children working towards mastery. If we are concerned about the rate at which children are mastering facts, we should be selective with the facts that children use for

the various learning activities, focusing children exclusively on the facts that they have not yet mastered. This can be done with small changes to the way in which we implement activities with the children.

Theme Three: Facts mastery progresses through predictable stages. Working with these stages allows us to isolate particular "difficulties", which also serve as cues to tell us when a child is ready to move on to another activity. The goal of each stage is as follows:

1. UNDERSTANDING CONCEPT: Introduce the concept of the operation. Put another way, the child must have a concrete understanding of what + means before learning addition facts. Otherwise, he is memorizing facts abstractly with nowhere to "file" them in his brain for ready-recall. Generally speaking, the concept is introduced with Golden Beads operations work.
   Addition is putting together
   (after attaining facility with + facts) Multiplication is repeated addition
   (after attaining facility with x facts) Subtraction is "take away", comparing and/or finding differences
   (after attaining facility with - facts) Division is sharing out evenly and/or repeated subtraction / making equal-sized groups
2. EXPLORATION: Explore randomly ordered math facts with materials to solidify the concept. Some facts may be internalized in the process, but it is not systematic.
3. RESEARCH: Systematically research math facts for a given operation with materials (discovering the tables)
4. REACH FOR AUTOMATICITY:
   - ASSESS: Understand of the goal of automaticity. Evaluate which facts are automatic and which require targeted practice.
   - NARROW: Teach "big concept" lessons to assist in abstraction & facts mastery
     - Commutative property for +, x
     - Reciprocity of + and -, of x and ÷
   - TARGETED PRACTICE: Engage in targeted practice of facts not-yet automatic. Use various materials / methods. Teach specific strategies for trickier facts.
     - Not for every fact, nor for every child (facts / fact families, patterns, memory aids)
5. REASSESS: Periodic assessment of progress (on a regular interval or when a child is ready)
6. MAINTAIN: Refresh with ongoing activities / episodic drills, and by helping others who work towards mastery.

Theme Four: Make every activity as multi-modal as possible
Remember the take-away from Part I of this monograph is that the more modalities we use to teach a new concept, the more neural pathways are built or activated and the greater the probability that the learner will retain the concept being explored or practiced.

Theme Five: Isolated Difficulty of each activity determines how long to stay on that activity
Often I am asked how long a child should remain with a given material or activity. If we maintain a laser focus on the isolated difficulty for each activity we will know what to watch for! Once the child has authentically mastered the isolated difficulty, he can move on!

Theme Six: Nightly FUN homework either reinforces foundational skills, practices / utilizes facts being mastered, OR acts as indirect preparation for what is to come.
Children who augment classroom facts work with as little as 10 minutes of homework each evening will achieve automaticity far more quickly (and possibly more thoroughly) than those who rely exclusively on classroom work. Each stage in the development of math fact automaticity suggests a particular type / focus for homework. Some homework lays foundational skill and knowledge. Some focuses on as yet unmastered facts. Some indirectly prepares children for what is to come. Tailoring homework to the child's math facts developmental level yields more rapid retention and more complete understanding.

Addition Math Facts

Concept: Addition Is Putting Quantities Together

The concept of addition is first encountered with Golden Beads. Two or three children retrieve specific teacher-chosen quantities and then, "see how much this is altogether".

Exploration: Combinations To Make 10

Combinations to 10 are explored first because they are key to children's mastery of sums that are greater than 10. For example, a child who really knows his combinations to 10 and is taught to find combinations to 10 within higher sums*, when confronted with 8+6, will initially think, "I know that 8 wants to be 10, so it grabs 2 from 6, leaving 4; 8+6 = 10+4 = 14. Saying the same thing with mathematical sentences:

8 + 6 =
8 + (2 + 4) =
(8 + 2) + 4=
10 + 4 = 14

**** This process of seeking combinations to 10 within higher sums will be discussed shortly.***

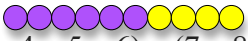
Without this kind of logic, memorization of 8 + 6 is relegated to "grill and drill" or making up mnemonic devices or other memory aides. While mnemonics and memory aides are fine for the last few tricky facts, we certainly would prefer that the child not have a trick for every math fact!

Material: Snake Game Addition – Even Snakes (Snakes to make 10s)

- ***Lesson notes:*** These problems are prepared ahead so that every sequential pair of bead bars sums to exactly 10. The isolated difficulty in this lesson is discovering and practicing addition facts with pairs of addends that sum to exactly 10.

6 + 4 + 5 + 5 + 3 + 7 + 1 + 9 + 8 + 2 =

Ideally, we want to use this lesson to transition the child from "counting all" to "counting up". In the beginning, a child will pull down the 6-bar and the 4-bar and count,



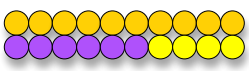
"(1 – 2 – 3 – 4 – 5 – 6) – (7 – 8 – 9 – 10)"

Ideally, the child will soon come to realize that she doesn't need to count the beads on the first bead bar; she knows that it is 6 because of the color. Now she can count,

"(6) – (7 – 8 – 9 – 10)"

NOTE: A common error in counting up is for a child erroneously to count the first bead in the second bar with the value of the first bar, so that his or her answers are consistently off by 1. (Here, that would manifest as the child counting the yellow beads, "6 – 7 – 8 – 9".) Be very precise in initial presentation. Place the counter at the end of the first bead bar and say, "Six. What comes next after 6?"

Making this activity multi-modal: This lesson is already very visual and kinesthetic. To extend the activity to cover auditory/oral processors, teach children to verbalize each combination before removing the colored bead bars.



"6 + 4 = 10"

After verbalizing this combination, put the 6 and 4 beads aside, leaving the golden 10-bar on the mat and bring down the next two bars.

The same concept of verbalizing combinations can be applied to work with the Number Rods.