# Reciprocity of Fraction Division and Multiplication
the path to abstraction

When children have had ample practice with dividing fractions and mixed numbers using materials and with abstract fraction multiplication, they may be ready to deduce the relationship between fraction division and multiplication.

MATERIALS:
Large Fraction Skittles
Cut-Out Labeled Fraction Circles
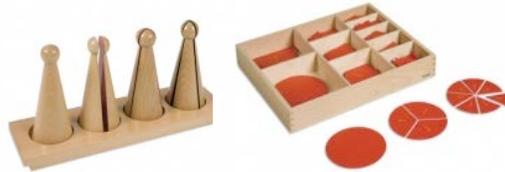
PREPARATION:
Ample practice with fraction and mixed operations using materials, especially division.
Ability to perform fraction and mixed number addition, subtraction and multiplication abstractly.
Facility with converting between improper fractions and mixed numbers abstractly
Knowledge of canceling in abstract fraction is helpful but not necessary

AGE: 10-12 +

DIRECT AIM:
Discover, through manipulation of concrete materials, the reciprocal relationship between fraction division and multiplication: dividing by a fraction = multiplication by that fraction's reciprocal.

INDIRECT AIM:
Reinforce the reciprocity of multiplication and division whether fractions, wholes, decimals.

PRESENTATION:
(Note: review processes / concepts are in <u>plain</u> font style while new concepts are <u>bold</u>)

1. Begin with a simple problem where both dividend and divisor have 1 in the numerator, such as $1/4 \div 1/2$ =.

2. Have the child do the division problem with the materials as usual;

   • Distribute the dividend to the skittles. (1/4 to 1/2)

   • The division, the sharing-out, is now finished; however, in division we always want to know what one whole gets, so we must now research the answer.

- To do that, we must retrieve the other 1/2 skittle and give it the same amount that the first 1/2 skittle received.
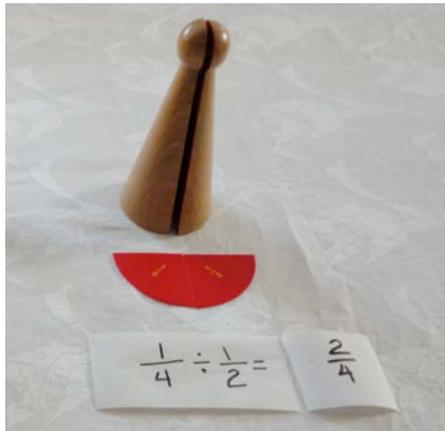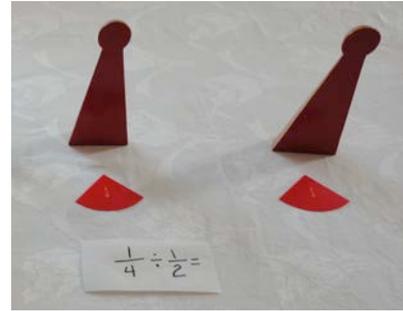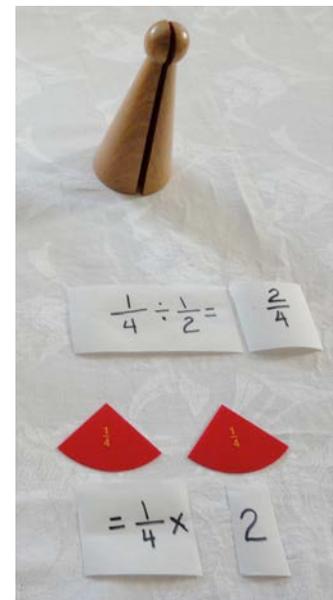

*If 1/2 receives 1/4, then 2/2 receive 2/4, or 1/2.*
$$1/4 \div 1/2 = 1/2$$

2) **Now analyze the process.**

3) **In our initial problem we had a dividend of 1/4.**
    How many dividends (fourths) did we have in our quotient (2)?
    Our quotient was one dividend taken <u>two times</u>.

4) **Thus 1/4 ÷ 1/2 =**
        1/4 X 2 = 2/4 = 1/2

5) Have the child repeat this process with problems of increased levels of difficulty such as
   - 2/5 ÷ 1/3 = (numerator of the divisor is 1)
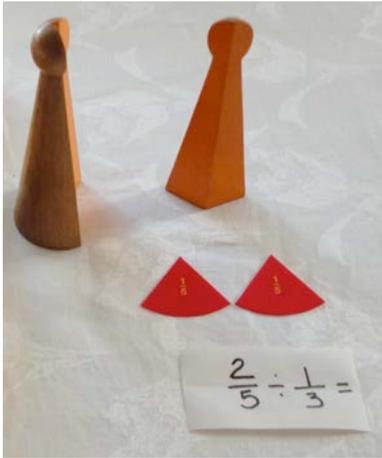
The division (sharing out) is done.

Now, we research the answer.
If 1/3 receives 2/5, how much does 3/3 receive?

One whole receives 6/5 or 1 1/5.
2/5 ÷ 1/3 = 6/5

The original dividend was 2/5.
Express the quotient in terms of the dividend:
How many sets of 2/5 are there in 6/5? (3)
2/5 ÷ 1/3 = 2/5 taken 3 times.
2/5 ÷ 1/3 =
2/5 x 3 = 6/5

- 3/8 ÷ 3/4 = (numerator of the divisor equals numerator of the dividend)
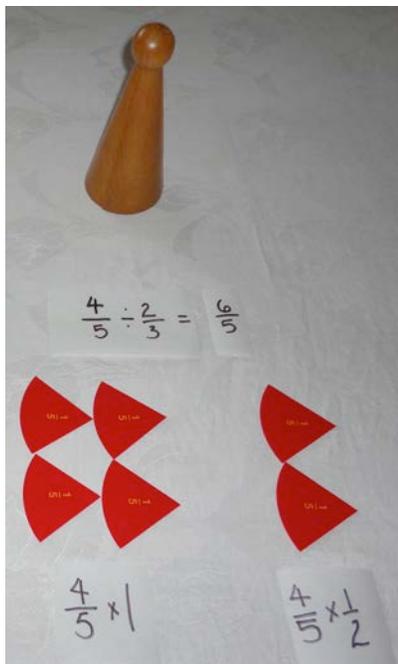- 4/5 ÷ 2/3 =   (numerator of dividend is divisible by the numerator of the divisor)

The division (sharing out) is done.

Now, we research the answer.
If 2/3 receives 4/5, each 1/3 receives 2/5.
How much does 3/3 receive? (6/5)

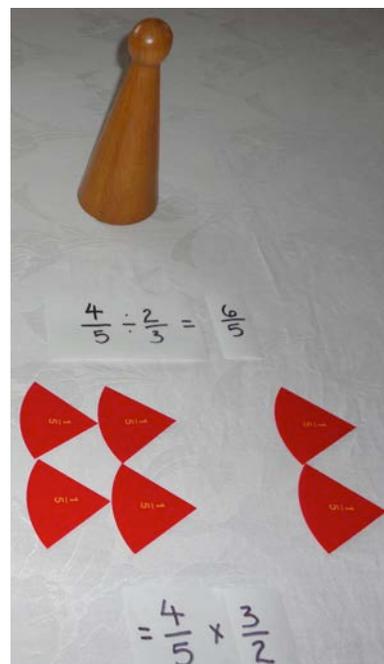
The original divisor was 4/5
Express the quotient in terms of the dividend:
How many sets of 4/5 are there in 6/5? (1½)
4/5 ÷ 2/3 = 4/5 taken 1½ times.

4/5 ÷ 2/3 =
4/5 x 3/2 = 6/5

6) Whether or not the child intuits the relationship of reciprocity from these problems, the following, more complicated types of problems should be done with materials in parallel to recording.   Always record <u>both</u> the division and multiplication problems with the quotient/product
   - 1/2 ÷ 2/3 = (numerator of dividend is not divisible by the numerator of the divisor)
   - 3 1/2 ÷ 2/3 = (problems using mixed numbers in the dividend)
   - 2/3 ÷ 3 1/2 = (problems using mixed numbers in the divisor)

7) Through this process, the child should garner the concept that fraction division problems can be converted to multiplication problems, and might even note that the divisor and multiplier are inverse of each other, reciprocals.